Photonic localization in one-dimensional k-component Fibonacci structures

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We studied the photonic localization of one-dimensional k-component Fibonacci structures (KCFS's), in which k different intervals are ordered according to a substitution rule. By using a transfer-matrix method, the optical transmission through KCFS's is obtained. It is demonstrated that the transmission coefficient has a rich structure, which depends on the wavelength of light and the number of different incommensurate intervals k. For the KCFS's with an identical k, by increasing the layer number of the sequences, more and more transmission dips develop and some of them approach zero transmission, which may finally make a one-dimensional photonic band gap. For a series of finite KCFS's, by increasing the number of different incommensurate intervals k, the total transmission over the spectral region of interest decreases gradually and the width of photonic band gap becomes larger. This property may be useful in the design of the high-performance optical and electronic devices. As for the infinite KCFS's, the transmission coefficient is singularly continuous and multifractal analysis is employed to characterize the transmission spectra. A dimensional spectrum of singularities associated with the transmission spectrum $f(\alpha)$ demonstrates that the light propagation in the KCFS's presents scaling properties and hence shows a genuine multifractality. [S0163-1829(98)10103-0]

I. INTRODUCTION

Since Anderson discussed the localization of electrons in disordered system in 1958,[1] the question of localization has been one of the most actively studied subjects in condensed-matter physics.[2] Recently, some fascinating issues have added fresh insight into the localization problem. First, the localization of states was recognized as a remarkable phenomenon that stems from the wave nature of the electronic states instead of only an electronic problem. Such localization is a feature related to any waves when there exists disorder in the structures, for example, it has also been reported in acoustic waves[3,4] and optical waves,[5,6] respectively. Second, the localized optical modes in certain dielectric microstructures have attracted much attention both theoretically and experimentally.[7–13] Since Yablonovitch and his coauthors have studied the propagation of electromagnetic waves in periodic dielectric media,[7–9] various dielectric materials have been exploited and the interest particularly lies in the dielectric structures possessing the photonic band gaps. A complete ''photonic band gap'' in a dielectric microstructure means the absence of photon propagation modes in any direction for a range of frequencies. Therefore, the studies of photonic band structures may have potential applications in optical and electronic devices. Third, the localization occurs not only in disordered system but also in the deterministic quasiperiodic system.[14–17] Furthermore, the exponentially localized states can also appear in other deterministic aperiodic systems such as the incommensurate Aubry-André model[18] and the deterministic aperiodic Rudin-Shapiro system.[19] It is well known that quasicrystals are perfectly ordered, but the Bloch theorem is inapplicable since there is no translational symmetry. On the other hand, the wave function is not exponentially localized as what happened in the disordered system. In some sense, the quasiperiodic system represents an intermediate case between periodic and disordered ones.

In one-dimensional (1D) quasiperiodic system, one of the well-known examples is the Fibonacci sequence. The Fibonacci sequence can be produced by repeated application of the substitution rule $A \rightarrow AB$ and $B \rightarrow A$, in which the ratio of the numbers of the two incommensurate intervals A and B is equal to the golden mean $\tau=(\sqrt{5}+1)/2$. Since Merlin *et al.* reported the realization of Fibonacci superlattices,[20] much attention has been paid to the exotic wave phenomena of Fibonacci systems in x-ray scattering spectra,[20–22] Raman scattering spectra,[23,24] and propagation modes of acoustic waves on corrugated surfaces.[25,26] However, the localization effect was not immediately apparent in these cases. In 1987 Kohmoto *et al.*[27] suggested that a suitable system for studying the photonic localization is classical electromagnetic waves in a quasiperiodic layered medium. Later the optical properties between Fibonacci and random multilayers were compared numerically[28] and optical transmission through binary multilayers arranged according to deterministic aperiodic distribution rules was investigated.[29] Very recently the experiments on the optical dielectric multilayers with Fibonacci structure were reported.[30] However, to the best of our knowledge, the localization problems of 1D aperiodic structures with more than two incommensurate intervals have not been studied so far, although their structural characterization and other physical properties have been performed previously.[31–35]

In this paper we report the photonic localization of 1D k-component Fibonacci structures (KCFS's), which contain k incommensurate intervals A_i $(i=1,2,\ldots,k)$ and can be generated by the substitute rule $A_1 \rightarrow A_1A_k$, $A_k \rightarrow A_{k-1}$, $\ldots$, $A_i \rightarrow A_{i-1}$, $\ldots$, $A_2 \rightarrow A_1$. By using a transfer matrix method, the optical transmissions through the k-component Fibonacci multilayers are calculated, which illustrates a rich structure. For the KCFS's with an identical k, when the layer number is large enough, one-dimensional photonic band gaps will appear in the transmission spectrum. Furthermore,

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as the number of different incommensurate intervals k increases, gradually wider photonic band gaps are exhibited in the spectra of the finite KCFS's. For the infinite KCFS's, the transmission coefficient is singularly continuous and multifractal analysis is employed to characterize these transmission spectra. It is known that multifractal analysis is a suitable statistical description of the long-term dynamical behavior of a physical system.[36,37] The multifractal formalism relies on the nonuniformity of the system. Our investigation demonstrates that the transmission spectra of the KCFS's are highly nonuniform intensity distributions that possess scaling properties of multifractal.

II. THEORETICAL MODEL AND NUMERICAL METHOD

Let us begin with the description of the k-component Fibonacci structures. First, we define a basis that includes k distinct incommensurate intervals $A_1, A_2, \ldots, A_k$. These intervals are arranged in a k-component Fibonacci sequence with a substitution rule S denoted as

$$A_1 \to A_1 A_k, \quad A_k \to A_{k-1}, \quad \ldots,$$

$$A_i \to A_{i-1}, \quad \ldots, \quad A_2 \to A_1.$$

In contrast, the KCFS's can be expressed by a limit of the generation of the sequence $C_n^{(k)}$. Let $C_n^{(k)} = S^n A_1$; thus

$$C_0^{(k)} = A_1,$$

$$C_1^{(k)} = A_1 A_k,$$

$$C_2^{(k)} = A_1 A_k A_{k-1},$$

$$\vdots$$

$$C_{k-1}^{(k)} = A_1 A_k A_{k-1} \cdots A_3 A_2$$

and in general $C_n^{(k)} = C_{n-1}^{(k)} + C_{n-k}^{(k)}$. If the interval number of the generation $C_n^{(k)}$ is defined as $F_n^{(k)}$, $F_n^{(k)}$ is satisfied by $F_n^{(k)} = F_{n-1}^{(k)} + F_{n-k}^{(k)}$, with $F_i = i+1$ $(i = 0,1,\ldots,k-1)$. We denote the number of A_i $(i = 1,2,\ldots,k)$ in $C_n^{(k)}$ as $N_n^{(k)}(A_i)$. The ratios of these numbers are defined as $\eta_i = \lim_{n\to\infty}[N_n^{(k)}(A_i)/N_n^{(k)}(A_1)]$. It turns out that the set $\{\eta_i\}$ satisfies

$$\eta_k^k + \eta_k = 1,$$

$$1:\eta_k = \eta_k:\eta_{k-1} = \cdots = \eta_i:\eta_{i-1} = \cdots = \eta_3:\eta_2. \quad (1)$$

Therefore, all these ratios $\eta_i = \eta_k^{k-i+1}$ $(1 < i \leqslant k)$ are irrational numbers between zero and unity except $\eta_1 = 1$. It has been proved[31] that the KCFS's are quasiperiodic when $1 < k \leqslant 5$, while for $k > 5$, the KCFS's are nonquasiperiodic, but they are still ordering.

The system we study here is the k-component Fibonacci (KCF) multilayers consisting of k different kinds of layers $A_1, A_2, \ldots, A_i, \ldots, A_k$ with indices of refraction $\{n_i\}$ and thicknesses $\{d_i\}$, respectively (where $i = 1,2,\ldots,k$). Now we consider the optical propagation through the KCF multilayers. In the case with normal incidence and polarization parallel to the multilayer surfaces, the transmission through the interface $A_j \leftarrow A_i$ is given by the transfer matrix

$$T_{j,i} = \begin{pmatrix} 1 & 0 \\ 0 & n_i/n_j \end{pmatrix} \quad (2)$$

and the light propagation within a layer A_i is described by a matrix T_i where

$$T_i = \begin{pmatrix} \cos\delta_i & -\sin\delta_i \\ \sin\delta_i & \cos\delta_i \end{pmatrix}, \quad (3)$$

where the phase δ_i is given by $\delta_i = g n_i d_i$, g is the vacuum wave vector, and d_i is the thickness of the layer A_i. Then the propagation of light through an aperiodically layered medium can be expressed by multiplying matrices of the different layers. For example, the transmission of light through a multilayer ordering as $\{A_i A_j A_m\}$ can be obtained by the matrix $M = T_m T_{m,j} T_j T_{j,i} T_i$.

Considering the experimental setup, the k-component Fibonacci multilayer $C_n^{(k)}$ is sandwiched between two media of material A_1, the corresponding transfer matrix is

$$M_n^{(k)} = M_{n-k}^{(k)} M_{n-1}^{(k)}, \quad (4)$$

where

$$M_0^{(k)} = T_1^{(k)}, \quad M_1^{(k)} = T_k^{(k)} T_{k,1}^{(k)} T_1^{(k)},$$

$$M_2^{(k)} = T_{k-1}^{(k)} T_{k-1,k}^{(k)} T_k^{(k)} T_{k,1}^{(k)} T_1^{(k)}, \ldots,$$

$$M_{k-1}^{(k)} = T_2^{(k)} T_{2,3}^{(k)} T_3^{(k)} T_{3,4}^{(k)} \cdots T_{k-1}^{(k)} T_{k-1,k}^{(k)} T_k^{(k)} T_{k,1}^{(k)} T_1^{(k)}.$$

Therefore, the whole multilayer is represented by a product matrix $M_n^{(k)}$ relating the incoming and reflected waves to the transmitted wave. From this expression the transmission coefficient can be written as

$$T[C_n^{(k)}] = \frac{4}{|M_n^{(k)}|^2 + 2}, \quad (5)$$

where $|M_n^{(k)}|^2$ denotes the sum of the squares of the four elements of $M_n^{(k)}$.

III. NUMERICAL RESULTS AND DISCUSSION

Based on Eqs. (4) and (5), the optical transmission through the KCFS's can be calculated. The indices of the refraction corresponding to the k different layers $\{A_i\}$ are chosen as $n_i = 3\eta_i$ in this whole optical investigation of the KCFS's, where η_i can be given by Eq. (1). In order to exhibit more clearly the effect of the underlying geometrical structures, we consider the simplest setting. We suppose that the index of refraction is wavelength independent and the thicknesses of the k different layers $\{d_i\}$ are chosen to give $n_i d_i = nd$, i.e., the optical phases corresponding to the k different layers are the same as $\delta_i = \delta$ $(i = 1,2,\ldots,k)$.

We have studied a series of the transmission spectra of the KCFS's by increasing the number of layers and by varying the number of incommensurate intervals k. As an example, Fig. 1 gives the transmission coefficient T as a function of the phase δ in the interval $[\pi, 2\pi]$ for the three-component

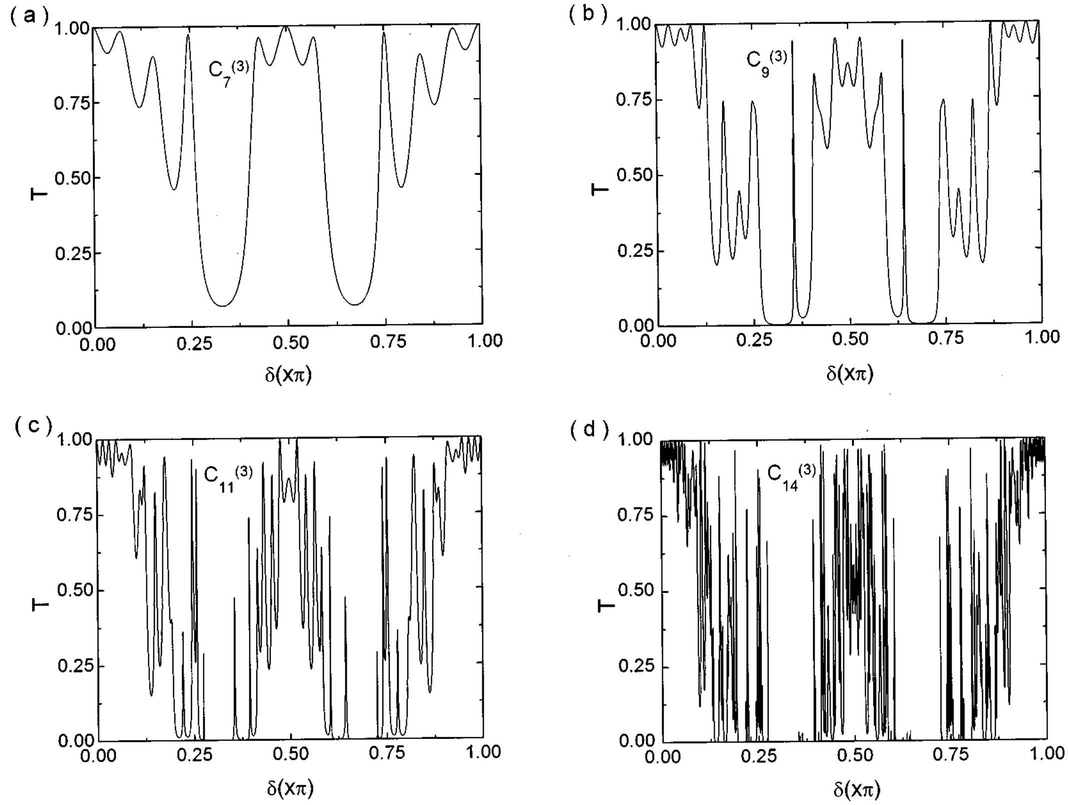

FIG. 1. Transmission coefficient T as a function of the phase δ for the three-component Fibonacci structures with the following generation and the number of layers: (a) $C_7^{(3)}$ and $N=13$, (b) $C_9^{(3)}$ and $N=28$, (c) $C_{11}^{(3)}$ and $N=60$, and (d) $C_{14}^{(3)}$ and $N=189$, respectively.

Fibonacci multilayers ($k=3$) with the generations $C_7^{(3)}$, $C_9^{(3)}$, $C_{11}^{(3)}$, and $C_{14}^{(3)}$, respectively, and the indices of refraction of the three different layers $\{A_i\}$ ($i=1,2,3$) are $n_1=3$, $n_2=1.397$, and $n_3=2.047$, respectively. It is evident that in the case of a very small number of layers there is no total reflection, although there exists some region of minimum transmission; when the number of layers becomes large, some regions may give rise to total reflection. Generally, by increasing the number of layers of the structures, more and more transmission zones diminish gradually and some of them approach zero transmission. In this way, a one-dimensional photonic band gap is realized. In order to have a quantitative impression, we define an ''average transmission'' as

$$\langle T\rangle_{ave}=\frac{1}{\pi}\int_{\pi}^{2\pi}T(\delta)d\delta. \tag{6}$$

It follows that the average transmissions of Figs. 1(a)–1(d) are $\langle T\rangle_{ave}\cong 0.640$, 0.519, 0.424, and 0.302, respectively. Therefore, the total transmission over the spectral region of interest definitely decreases when the number of layers in the KCFS's (k is fixed) increases due to the appearance of photonic band gaps.

It is also enlightening to compare the optical propagation behaviors of the KCFS's with different number of incommensurate intervals k. The calculations are performed on the transmission of different KCFS's with almost identical numbers of layers. Figure 2 illustrates the transmission coefficient T as a function of the phase δ for four KCFS's with different k. It can be easily seen that with increasing k, the photonic band gaps can be easily observed. Meanwhile the average transmission defined above varies as $\langle T\rangle_{ave}\cong 0.499$, 0.268, 0.151, and 0.127, corresponding to Figs. 2(a), 2(b), 2(c), and 2(d), respectively. Hence the total transmission over the spectral region decreases gradually. Consequently, wider photonic band gaps appear when k increases in the KCFS's. Moreover, when the number of layers in the KCFS's is sufficiently large, the width of the photonic band gap in the corresponding transmission spectra increases significantly when k increases. This tendency is demonstrated clearly in Figs. 3(a)–3(d), where the number of layers is about $N=30\,000$. It is well known that the existence of the photonic band gap is of great interest for potential technological applications. The overlap of the photonic gap and electronic band edge suppresses the spontaneous emission of light and favors the population reverse, which can improve the performances of many optical and electronic devices.[7–9] Obviously, the large photonic band gap of the dielectric structures may make it easier to satisfy the technical requirements. From this point of view, the KCFS's might be a kind of structural design for the high-performance optical and electronic devices.

In addition, it is interesting to investigate the optical features of the KCFS's with $k>5$, which actually belong to nonquasiperiodic structures. As an example, Figs. 4(a)–4(d) demonstrate the transmission coefficient T as a function of the phase δ for the KCFS's with $k=6$ and 10, respectively. The average transmission varied as $\langle T\rangle_{ave}\cong 0.1034$, 0.0278, respectively, in Figs. 4(a) and 4(b) corresponding to $k=6$, and $\langle T\rangle_{ave}\cong 0{,}0556$, 0.0126, respectively, in Figs. 4(c) and 4(d), corresponding to $k=10$. One may find that the electro-

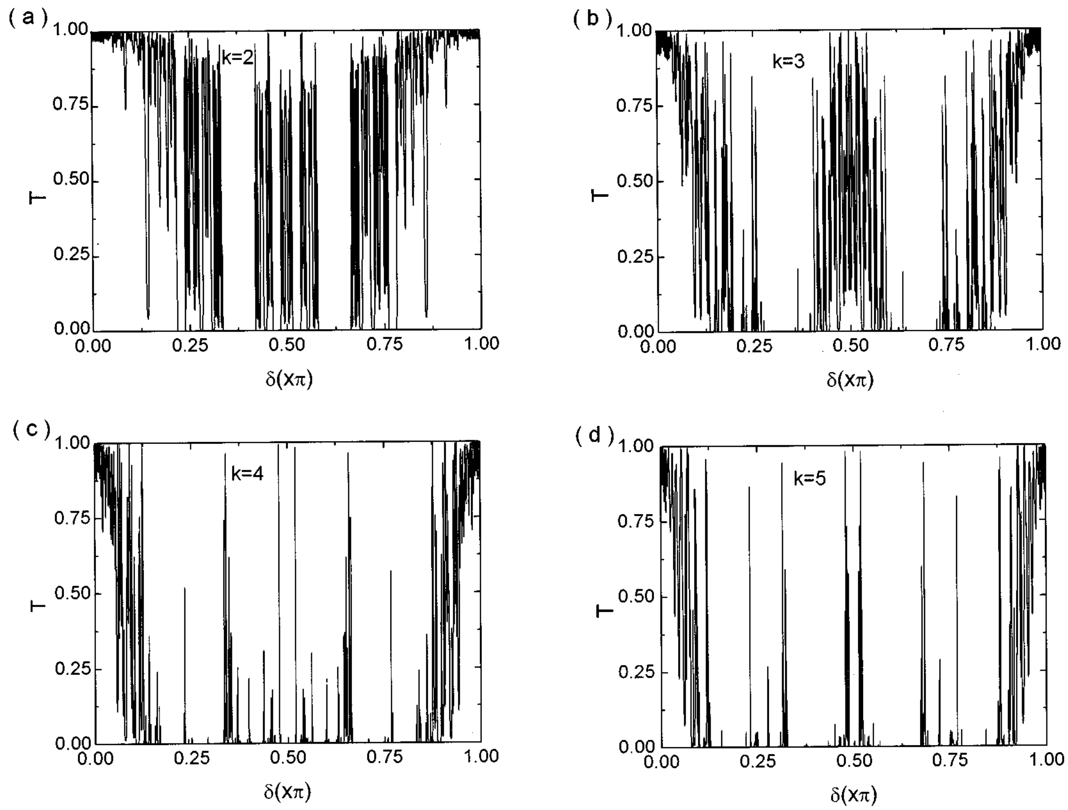

FIG. 2. Transmission coefficient T as a function of the phase δ for the k-component Fibonacci structures with the different incommensurate intervals k. The value of k, the generation, and the number of layers N are as follows: (a) $k=2$, $C_{12}^{(2)}$, and $N=233$; (b) $k=3$, $C_{15}^{(3)}$, and $N=277$; (c) $k=4$, $C_{17}^{(4)}$, and $N=250$; and (d) $k=5$, $C_{19}^{(5)}$, and $N=245$, respectively.

magnetic waves are also localized in these nonquasiperiodic structures and the photonic band gaps in these transmission spectra develop when the number of layers is increased. These features resemble the results from the Rudin-Shapiro system.[19] Moreover, with increasing k, the width of photonic band gaps enlarges and the optical behaviors in the KCFS's with $k>5$ are much closer to those of random distributions in some senses. Further studies on this aspect are being undertaken.

IV. SCALING PROPERTIES IN THE TRANSMISSION SPECTRA OF THE KCFS's

In the previous discussion we applied the average transmission to describe the whole transmission spectra of the KCFS's. This analysis, however, is limited especially in the case when the number of layers in the KCFS's is large enough. Actually, the transmission spectra shown in Figs. 3(a)–3(d) should be neither discrete nor continuous. These complicated spectra can be characterized by statistical methods such as multifractal analysis. Multifractal analysis is a tool for characterizing the nature of a positive measure in a statistical sense.[38–41] If a positive measure is covered with boxes of size ε and $p_i(\varepsilon)$ is denoted as the probability (integrated measure) in the ith box, an exponent (singularity strength) α_i can be defined as

$$p_i(\varepsilon)\sim\varepsilon^{\alpha_i}. \tag{7}$$

If we count the number of boxes $N(\alpha)d\alpha$ where the probability p_i has singularity strength between α and $\alpha+d\alpha$, then $f(\alpha)$ can be loosely defined as the fractal dimension of the set of boxes with singularity strength α, that is,

$$N(\alpha)d\alpha\sim\varepsilon^{-f(\alpha)}d\alpha. \tag{8}$$

The $f(\alpha)$ singularity spectrum provides a mathematically precise and intuitive description of the nonuniform systems. On the other hand, it should be mentioned that the generalized dimension D_q provides an alternative description of the singular measure. It is defined as

$$D_q=\frac{1}{q-1}\lim_{\varepsilon\to 0}\frac{\ln\Sigma_i[p_i(\varepsilon)]^q}{\ln\varepsilon}. \tag{9}$$

D_q corresponds to scaling exponents for the qth moments of the measure.

In the case of the transmission spectrum, the optical transmission coefficient is a positive quantity and the phase space is a support. A straightforward application of the multifractal formalism requires the evaluation of the exact integral of the intensity measure of the structures with infinite length over a small segment of length in the phase space. In this case, the computer time for calculation will increase incredibly. To solve this problem, an approximate scheme is chosen.[41] Instead of calculating the infinite KCFS's, we only deal with a structure that contains repeating copies of finite generation, i.e., $C_n^{(k)}$ of the original structure. It is known that the transmission of a periodic multilayer is also periodic and the pe-

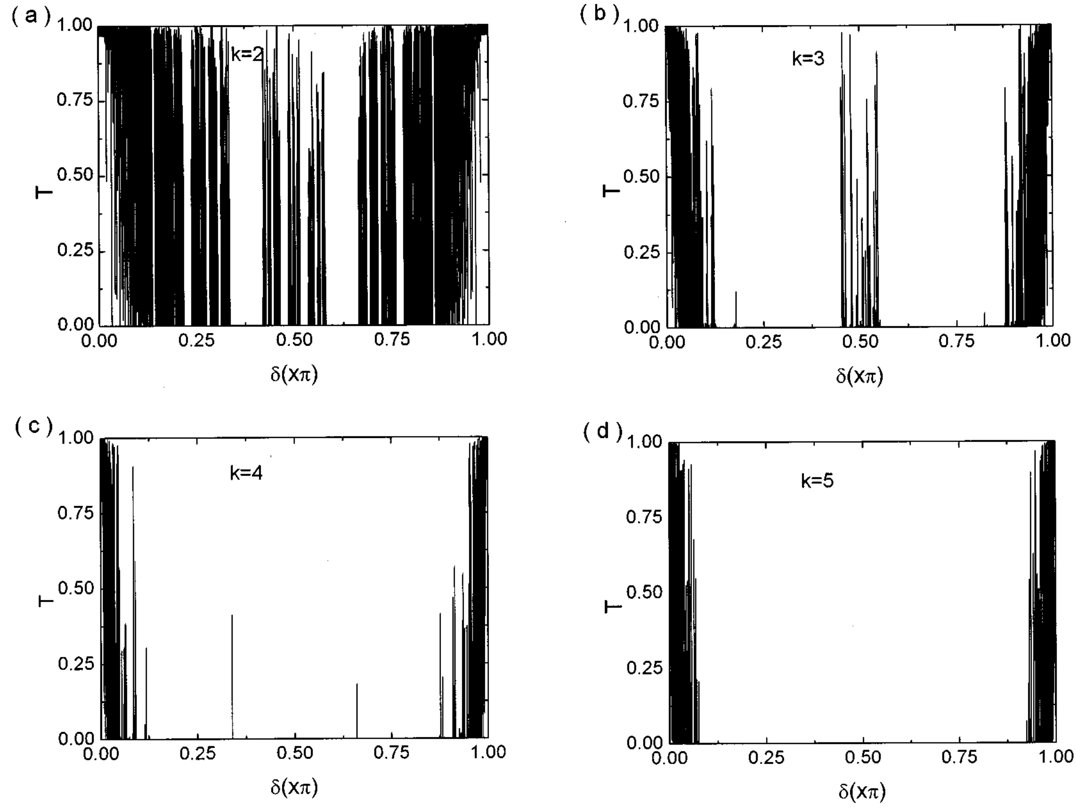

FIG. 3. Transmission coefficient T as a function of the phase δ for the k-component Fibonacci structures with the different incommensurate intervals k. The value of k, the generation, and the number of layers N are as follows: (a) $k=2$, $C_{22}^{(2)}$, and $N=28657$; (b) $k=3$, $C_{27}^{(3)}$, and $N=27201$; (c) $k=4$, $C_{32}^{(4)}$, and $N=31422$; and (d) $k=5$, $C_{36}^{(5)}$, and $N=29244$, respectively.

riodicity is π. Therefore, we need only consider the situation in one period of phase space. The essential ingredient in multifractal characterization is the probability weights p_i. In our case, p_i is denoted as the weight of the transmission coefficient in the transmission spectrum, i.e.,

$$p_i=\frac{|T_i|^2}{\sum_{i=1}^{N}|T_i|^2}, \tag{10}$$

where T_i is the transmission coefficient [shown in Eq. (5)] with the phase $\delta_i=\pi(i/N)$ $(i=1,2,\ldots,N)$ and the number of layers $N=F_n^{(k)}$. The partition function can then be expressed as

$$Z(q)=\sum_{i=1}^{N} p_i^q,$$

$$Z'(q)=\frac{dZ}{dq}=\sum_{i=1}^{N} p_i^q \ln p_i, \tag{11}$$

$$Z''(q)=\frac{d^2Z}{d^2q}=\sum_{i=1}^{N} p_i^q (\ln p_i)^2,$$

where the parameter q provides a ''microscope'' for exploring the singular measure in different regions. For $q>1$, $Z(q)$ amplifies the more singular regions of p_i, while for $q<1$ it accentuates the less singular regions. For $q=1$ the measure $Z(1)$ replicates the original measure. The $f(\alpha)$ curve of any finite sample is therefore available at a local level, i.e., for a given phase space. The values of α and $f(\alpha)$ are given by

$$\alpha=-\frac{Z'(q)}{Z(q)\ln N},$$

$$f(\alpha)=\frac{1}{\ln N}\left(\ln Z(q)-\frac{qZ'(q)}{Z(q)}\right). \tag{12}$$

The generalized dimensions D_q are related to the spectrum of singularity $f(\alpha)$ by the Legendre transform

$$f(\alpha)=\alpha q-(q-1)D_q,$$

$$\alpha(q)=\frac{d}{dq}(q-1)D_q. \tag{13}$$

In order to illustrate the multifractality of the transmission spectra of the KCFS's shown in Figs. 3(a)–3(d), we calculate the corresponding $f(\alpha)$ spectra shown in Fig. 5(a) according to Eqs. (11)–(13). In Fig. 5(a) the data points fit perfectly into a smooth curve, which is a characteristic of an infinite structure. The quantity $f(\alpha)$ is commonly the dimension of the set of phases δ in the transmission spectrum. In particular, there are several physical meanings in the $f(\alpha)$ spectrum of a transmission measure. (i) The abscissa α_0 of the summit of the $f(\alpha)$ curve, which corresponds to $q=0$, is the strength of a generic singularity. In some senses, the exponent α_0 characterizes the behavior of the transmission at a generic singularity. Obviously $f(\alpha_0)<1$, which means that the support of the transmission is not the whole δ axis due to the

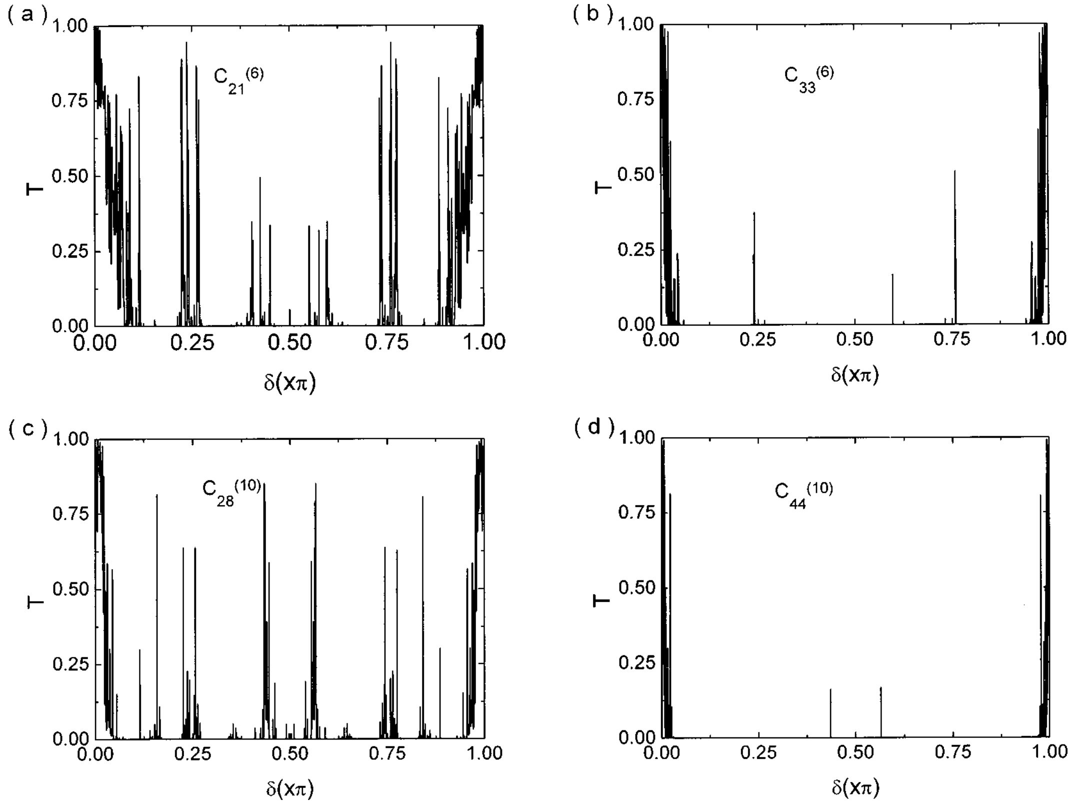

FIG. 4. Transmission coefficient T as a function of the phase δ for the k-component Fibonacci structures with $k>5$. The value of k, the generation, and the number of layers N are as follows: (a) $k=6$, $C_{21}^{(6)}$, and $N=251$; (b) $k=6$, $C_{33}^{(6)}$, and $N=5103$; (c) $k=10$, $C_{28}^{(10)}$, and $N=265$; and (d) $k=10$, $C_{44}^{(10)}$, and $N=4746$.

existence of the optical band gaps in the spectra. Moreover, because the width of the optical band gap increases when k propagates, the fractal dimension of the support $f(\alpha_0)$ decreases correspondingly. (ii) The extremes α_{min} and α_{max} of the abscissa of the $f(\alpha)$ curve represent the minimum and the maximum of the singularity exponent α, which acts as an appropriate weight in phase space. In fact, $\alpha_{min} = \lim_{q\to+\infty} D_q$ and $\alpha_{max} = \lim_{q\to-\infty} D_q$ characterize the scaling properties of the most concentrated and most rarefied region of the intensity measure, respectively. By increasing the number of incommensurate intervals k in the KCFS's, the value of $\Delta\alpha = \alpha_{max} - \alpha_{min}$ also gradually increases. This implies that the optical transmission measure of the KCFS's approaches the behavior of a random system when k increases. (iii) The dimension of the set of transmission peaks $d_p = f(1)$, corresponding to $\alpha = 1$. d_p represents the dimension of the set of phase δ for which the local singularity exponent α is less than unity. In Fig. 5(a) we have $d_p < 1$; when k increases, d_p decreases evidently. Therefore, different KCFS's exhibit different transmission distributions.

The generalized dimension D_q characterizes the nonuniformity of the measure, positive q's accentuate the denser regions, and negative q's accentuate the rarer ones. Figure 5(b) shows the plot of generalized dimension D_q vs q for the transmission spectra of the KCFS's shown in Fig. 3. The plots of D_q vs q in Fig. 5(b) correspond to the plots of $f(\alpha)$ vs α in Fig. 5(a). For some special values of q, one can take D_q as the dimension of a special set, which supports a particular part of the measure. (i) D_0 for $q=0$, i.e., $D_0 = \lim_{\varepsilon\to 0}[\ln N(\varepsilon)/\ln(1/\varepsilon)]$, where $N(\varepsilon)$ is the number of line segments of size ε to cover the whole phase axis, is the dimension of the support as mentioned above, $D_0 = f(\alpha_0) < 1$. (ii) D_1 for $q\to 1$ is the information dimension of the intensity measure,

$$D_1 = \lim_{\varepsilon\to 0} \frac{-\Sigma_i p_i(\varepsilon)\ln p_i(\varepsilon)}{\ln(1/\varepsilon)},$$

where $-p_i(\varepsilon)\ln[p_i(\varepsilon)]$ is an expression from information theory and corresponds to the amount of information associated with the distribution of $p_i(\varepsilon)$ values. For $q=1$, $f(\alpha(1)) = \alpha(1) = D_1$. The distance of D_1 to unity is a faithful measure of how singular the transmission measure is. Figure 5(b) shows that the information dimension D_1 in the KCFS's is less than the dimension of the support D_0, i.e., $D_1 < D_0 < 1$. So the transmission distribution of the KCFS's with $2 \leqslant k \leqslant 5$ is definitely a fractal measure. (iii) D_2 for $q=2$ is the correlation dimension,

$$D_2 = \lim_{\varepsilon\to 0} \frac{\ln\Sigma_i p_i^2(\varepsilon)}{\ln\varepsilon} = \lim_{\varepsilon\to 0} \frac{\ln\langle\mu(\varepsilon)\rangle}{\ln\varepsilon},$$

where $\langle\mu(\varepsilon)\rangle$ is the average density of the transmission peaks in the phase interval of $\varepsilon = \Delta\delta$ in the transmission measure of the KCFS. We have $D_2(k) < D_2(k')$ in the KCFS's if $k > k'$ (for example, $D_2 \approx 0.77$ for $k=3$ and $D_2 \approx 0.69$ for $k=5$). It has been demonstrated that when k be-

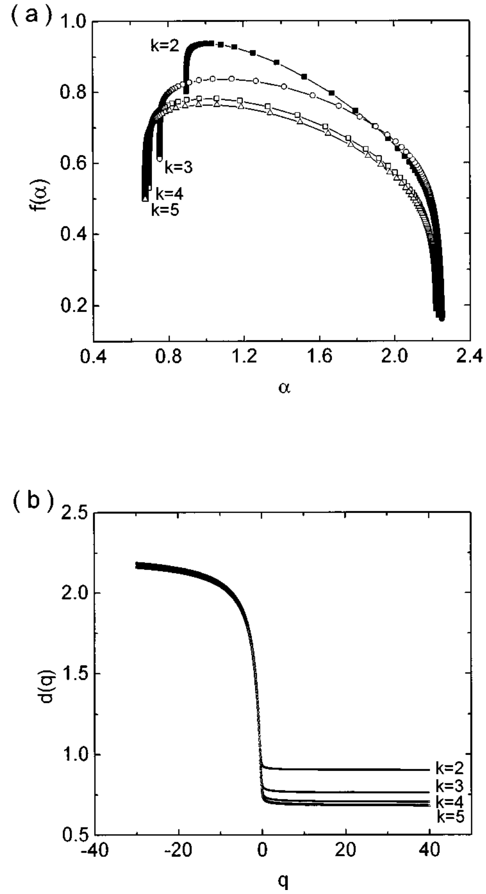

FIG. 5. (a) $f(\alpha)$ spectra and (b) plot of the generalized dimension D_q as a function of q, for the transmission distributions of the KCFS's where $k=2,3,4,5$, respectively.

comes larger, there are fewer transmission peaks occurring in the transmission spectrum of the KCFS's and the optical band gaps are definitely enlarged.

The above scaling analysis indicates that the transmission spectra of the KCFS's $(2 \leqslant k \leqslant 5)$ are singular continuous and possess multifractality. When k increases, the photonic localization of the electromagnetic wave is exhibited and wider optical band gaps are found.

V. CONCLUSION

In this paper we have presented the transmission of electromagnetic wave through the k-component Fibonacci structure, which contains k different incommensurate intervals and can be generated by a substitution rule. The transmission spectra of the k-component Fibonacci multilayers have been obtained by a transfer matrix method. It has been demonstrated that the transmission coefficient has a rich structure: For the KCFS's with a fixed k, the photonic localization is expected and the one-dimensional band gap appears when the layer of the sequence becomes sufficiently large; on the other hand, for the finite KCFS's with gradually increasing k, the width of the photonic band gap becomes larger. These interesting properties make the KCFS's a possible candidate of the designed material for the high-performance optical and electronic devices. When the number of layers approaches infinity, the transmission coefficient is expected to demonstrate a multifractal behavior. Multifractal analysis reveals that these transmission measures can be characterized by a monotonically decreasing dependence of D_q vs q; the dimension spectrum of singularities $f(\alpha)$ is a smooth function with a summit of $D_0 < 1$. The transmission measure does not have an absolutely continuous component. Therefore, the optical propagations through the KCFS's $(2 \leqslant k \leqslant 5)$ are singular continuous and possess multifractal properties. Finally, an experimental investigation of the transmission through the media of the KCFS's is expected in further study.

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