Symmetry-induced perfect transmission of light waves in quasiperiodic dielectric multilayers

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Resonant transmission of light has been observed in symmetric Fibonacci TiO_2/SiO_2 multilayers, which is characterized by many perfect transmission peaks. The perfect transmission dramatically decreases when the mirror symmetry in the multilayer structure is deliberately disrupted. Actually, the feature of perfect transmission peaks can be considered as general evidence for dielectric multilayers with symmetric internal structure. It opens a unique way to control light propagation. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1468895]

Motivated by the pioneering work of Yablonovitch[1] and John,[2] much attention has been paid to photonic crystals, which forbid the propagation of photons with a certain range of energies known as the photonic band gaps (PBGs).[3] Nowadays, great effort is being put in the manipulation of photonic energy bands (PBG) in crystals in order to make photons become a real alternative to electrons as information carriers.[4] One of the fundamental points in controlling the propagation of a lightwave is how to select photons of certain frequencies and to obtain a high transmittivity at those desired frequencies. Thereafter, the studies on PBG have been extended to include quasiperiodic photonic structures. Compared to the periodic structures, more structural parameters can be tuned in the quasiperiodic designs, thus opening a way to a wide range of technological applications in several different fields.[5]

The Fibonacci sequence is one of the well-known examples of one-dimensional (1D) quasiperiodic structures. The first Fibonacci superlattice was produced by Merlin *et al.*[6] in 1985. Since then, a considerable interest has been focused on the exotic wave phenomena of Fibonacci systems.[7–9] In 1987, Kohmoto, Sutherland, and Iguchi proposed the photonic Fibonacci multilayers.[10] Later, interesting issues related to optical propagation in various quasiperiodic structures have been investigated.[11–14] Most of the studies concentrated on photonic localization, and the intensity of the transmission peak was not well studied in previous research. However, these items are important for applications in optical communication. It is noteworthy that the transport of electrons in a 1D random-dimer model exhibits a localization–delocalization transition.[15] Physically, the extended electronic states in this system are due to the symmetry of its internal structure.[16] In this letter, we report an optical property of Fibonacci TiO_2/SiO_2 multilayers with internal symmetry.

The symmetric Fibonacci sequence can be generated in the following way. The jth generation of the sequence can be expressed as $S_j=\{G_j,H_j\}$, where G_j and H_j are Fibonacci sequences. G_j and H_j obey the recursion relations

$$G_j=G_{j-1}G_{j-2},\quad H_j=H_{j-2}H_{j-1},$$

with $G_0=H_0=B$ and $G_1=H_1=A$. Therefore,

$$S_j=G_{j-1}G_{j-2}H_{j-2}H_{j-1}. \tag{1}$$

The system presented here is a symmetric Fibonacci multilayer (SFM) obtained with two different materials A and B, with refractive index $\{n_i\}$ and thickness $\{d_i\}$, respectively, $(i=A,B)$. The SFM with generation S_j has $2F_j$ dielectric layers, and the Fibonacci number $F_j=F_{j-1}+F_{j-2}$ (for $j>2$) with $F_0=0$ and $F_1=1$. Now we consider the optical propagation through the SFM. In the case of normal incidence and polarization parallel to the multilayer surfaces, the transmission through the interface $A\leftarrow B$ is given by the transfer matrix

$$T_{AB}=T_{BA}^{-1}=\begin{bmatrix}1 & 0\\ 0 & r\end{bmatrix}, \tag{2}$$

where $r=n_B/n_A$. The light propagation within a layer A (or B) is described by a matrix $T_{A(B)}$:

$$T_{A(B)}=\begin{bmatrix}\cos\delta_{A(B)} & -\sin\delta_{A(B)}\\ \sin\delta_{A(B)} & \cos\delta_{A(B)}\end{bmatrix}, \tag{3}$$

where the phase $\delta_{A(B)}$ is given by $\delta_i=kn_id_i$ ($i=A$ or B), and k is the vacuum wave vector. Then, the whole multilayer is represented by a product matrix M_j relating the incoming and reflected waves to the transmitted wave. The total transmission matrix of S_j has the form

$$M_j=\begin{bmatrix}m_{11}(j) & m_{12}(j)\\ m_{21}(j) & m_{22}(j)\end{bmatrix}. \tag{4}$$

Considering the symmetry in the structure [shown in Eq. (1)] and using the unitary condition $\det|M_j|=1$, the transmission coefficient of the SFM can be written as

$$T[S_j]=\frac{4}{|M_j|^2+2}=\frac{4}{[m_{12}(j)+m_{21}(j)]^2+4}, \tag{5}$$

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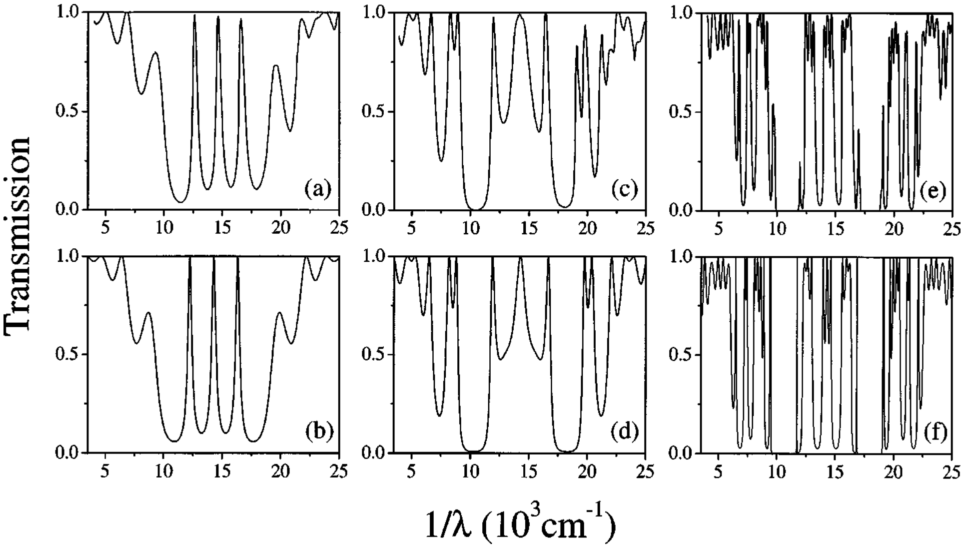

FIG. 1. Measured and calculated transmission coefficient T vs wave number λ^{-1} for symmetrical Fibonacci TiO_2/SiO_2 multilayers (SFMs) with the different generations S_j. SFM S_5 with 16 layers: (a) measured and (b) calculated; SFM S_6 with 26 layers: (c) measured and (d) calculated; SFM S_8 with 68 layers: (e) measured and (f) calculated, respectively.

where $|M_j|^2$ denotes the sum of the squares of the four elements of M_j. Then, if the condition $m_{12}(j)+m_{21}(j)=0$, is satisfied, perfect transmission peaks are indeed obtained. It should be pointed out that Eq. (5) is generally applicable to any kinds of optical multilayers with a mirror symmetry.

It is well known that the transfer matrix of a Fibonacci sequence has a six-cycle property around $\delta_A=\delta_B=\delta=(m+1/2)\pi$, where m is an integer.[10] For a symmetric Fibonacci sequence, the equation for the transmission matrix M_j can also be considered as a dynamical map from the point of view of a renormalization group.[17] Around $\delta_A=\delta_B=\delta=(m+1/2)\pi$, we have $M_j=(-I)^{F_j}$ $(j=0,1,2,\dots)$, where I is the unit matrix and F_j is the Fibonacci number. Namely, there is a three-cycle property at $\delta=(m+1/2)\pi$. And the perfect transmission is expected since the transmission matrix is I or $-I$. This property implies that the transmission coefficient of the SFM has a type of self-similarity, i.e., $T(S_j)=T(S_{j+3})$.

In the experiments, we chose titanium dioxide and silicon dioxide as dielectric materials A and B, respectively. Around the wavelength of 700 nm, their refractive indices are $n_A=2.30$ and $n_B=1.46$, respectively. The multilayer films were fabricated by electron-gun evaporation on a glass substrate. Before the evaporation, the pressure of the chamber was lower than 2×10^{-5} Torr. The films were formed under an oxygen atmosphere: the pressure is 2×10^{-4} Torr for TiO_2 deposition, and 0.8×10^{-4} Torr for SiO_2. The film thickness was controlled by quartz-crystal monitoring at a frequency of 5 MHz, and also the quarter-wave and half-wave optical thicknesses were optically monitored at 700 nm. For the sake of simplification, the thicknesses of two materials in our films were chosen to satisfy the condition $n_Ad_A=n_Bd_B$, which gives the same phase shift in the two materials, i.e., $\delta_A=\delta_B=\delta$. Finally, the central wavelength was set to 700 nm or so, which gives $d_A=(700\text{ nm})/4n_A\cong76.1$ nm, and $d_B=(700\text{ nm})/4n_B\cong120.0$ nm.

The optical transmission of these SFM films has been investigated. The transmission spectra were measured by using a U-3410 spectrophotometer in the range of wavelength from 185 to 2600 nm. Figure 1 shows the experimentally measured and the theoretically calculated transmission coefficients as a function of wave number for SFM films with generations S_5, S_6, and S_8, respectively. It is evident that in the case of a small number of layers, regions of minimum transmission appears, which may eventually give rise to total reflection as the number of layers increases. In general, by increasing the number of layers, the quasiperiodicity becomes stronger, and more and more transmission zones die out gradually and some of them eventually approach zero transmission. In this way, a 1D photonic band gap is finally obtained, as happens in the cases of other quasiperiodic structures. Figure 1 indicates that the measured transmission peaks are in good agreement with the numerical calculations.

On the other hand, it is important to compare the optical behavior of the SFM with those of Fibonacci SiO_2/TiO_2 multilayers (without symmetry).[12] (Note that in these multilayers, the optical transmission does not change if the dielectric materials A and B are exchanged.) Unlike the poor transmission of the optical wave usually observed in ordinary Fibonacci multilayers,[12] in the SFMs perfect transmission peaks have been obtained experimentally (as shown in Fig. 1). More resonant modes with perfect transmission occur in

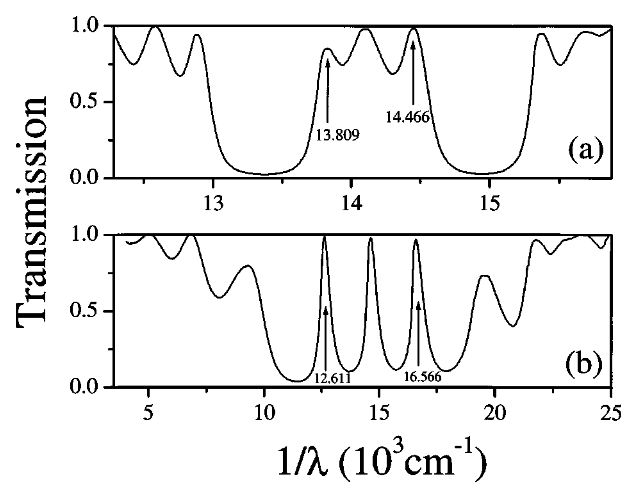

FIG. 2. Measured transmission coefficient T as a function of wave number λ^{-1} around $\lambda^{-1}=1.43\times10^4$ cm^{-1}, i.e., $\delta=0.5\pi$ for SFMs: (a) S_8 with 68 layers; and (b) S_5 with 16 layers, respectively.

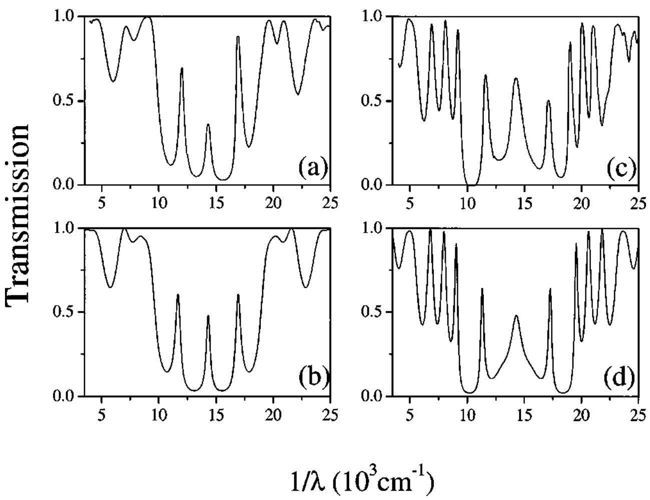

FIG. 3. Measured and calculated transmission coefficient T as a function of wave number λ^{-1} for disrupted symmetric Fibonacci TiO_2/SiO_2 mutilayers. In the structure S_5 the nineth layer and tenth layers are interchanged: (a) measured and (b) calculated; in the structure S_6 the 14th and 15th layers are interchanged: (c) measured and (d) calculated, respectively.

the gaps of a 1D photonic quasicrystal. The reason is that the mirror symmetry in the system may create special structures in some transfer matrix elements which make it easier to satisfy the condition of perfect transmission.

It is also enlightening to illustrate the scaling properties of the transmission spectra of SFMs. Figures 2(a) and 2(b) show the measured transmission coefficients around $\delta = \pi/2$ of the SFM with the generations S_8 and S_5, respectively. According to Fig. 2, it is easy to obtain the scaling factor $f=(16.566-12.611)/(14.466-13.809)\cong 6.02$, which is close to the analytical prediction $f=5.11$. The discrepancy arises from the fact that the number of layers in the multilayer film is limited while the theoretical result corresponds to infinite structures.

In order to confirm that the perfect transmission of light through the multilayers is induced by the symmetry of the internal structure, we deliberately disrupted the symmetry in the studied systems and checked the change of the transmission spectra. Figures 3(a) and 3(b) give the measured and the calculated transmission spectra of the resulting TiO_2/SiO_2 multilayers with generation S_5, respectively, where the ninth and tenth layers are interchanged. By comparing these results with the spectrum in Fig. 1(a), it appears that the previous perfect transmission peaks have clearly shrunk. And if the 14th and 15th layers are interchanged in generation S_6, the optical behavior [shown in Figs. 3(c) and 3(d)] also changes dramatically with the loss of the perfect transmission peaks in Fig. 1(c). Therefore, the phenomenon of perfect transmission in dielectric multilayers is indeed related to the symmetry of their internal structure.

In summary, we have investigated the optical properties of quasiperiodic SFMs of TiO_2/SiO_2. For normal-incidence light, many perfect transmission peaks have been observed, and if the symmetry is disrupted, these peaks clearly shrink. It has been demonstrated that the mirror symmetry plays an important role in obtaining the perfect transmission features in SFMs. This phenomenon can be regarded as a generic feature of multilayer films possessing an internal symmetry. Due to the transmittivity of resonant optical modes and to the rich structure of transmission spectra, the SFM films are suitable for a wide range of potential applications where the control of the propagation of light waves is a crucial requirement. We expect that this work will also contribute to the multiwavelength narrow-band optical filters, the wavelength division multiplexing system, and photonic integrated circuits, where high-transmission and high-resolution monofrequency outputs are particularly desired.

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