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# STRUCTURAL SYMMETRY AND OPTICAL PROPERTIES OF DIELECTRIC MULTILAYERS

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We have investigated the structural symmetry and optical properties of the dielectric multilayers. By using the transfer-matrix method, the propagation of electromagnetic wave in the dielectric multilayer film is obtained. It is shown that if a mirror symmetry is induced to the structure, perfect transmissions will definitely happen. And the perfect transmission can be controlled at certain wavelengths if the special structure with symmetry is achieved. Experimental observations are in good agreement with the theoretical predictions. This finding will have potential applications to optoelectric devices, such as the wavelength division multiplexing (WDM) system.

Since Yablonovitch[1] and John[2] studied the propagation of electromagnetic waves in periodic dielectric media, there has been increasing interest in studies of dielectric structures with the photonic band gaps, i.e. photonic crystals[3] or photonic quasicrystals.[4] These artificial materials are opening the way to control the propagation of a light wave and achieving a potential application on high-performance optical and electronic devices. In this paper, we report on the optical property of the dielectric multilayer with internal symmetry. It is found that the structural symmetry has an important influence on optical propagation of electromagnetic waves through the multilayer film.

Consider the optical propagation through the dielectric multilayer which consists of $m$ kinds of layers $A_1, A_2, \ldots, A_i, \ldots, A_m$ with indices of refraction $\{n_i\}$ and thicknesses $\{d_i\}$, respectively. In the case with normal incidence and polarization parallel to the multilayer surfaces, the transmission through the interface $A_j \leftarrow A_i$ is given by the transfer matrix[5]

$$T_{ji} = \begin{pmatrix} 1 & 0 \\ 0 & n_i/n_j \end{pmatrix} \qquad (1)$$

and the light propagation within a layer $A_i$ is described by a matrix $T_i$:

$$T_i = \begin{pmatrix} \cos \delta_i & -\sin \delta_i \\ \sin \delta_i & \cos \delta_i \end{pmatrix}, \qquad (2)$$

where the phase $\delta_i$ is given by $\delta_i = g n_i d_i$, $g$ is the vacuum wave vector, and $d_i$ is the thickness of the layer $A_i$. Therefore, the whole multilayer is represented by a product matrix $M$ relating the incoming and reflected waves to the transmitted wave. The total transmission matrix $M$ has the form

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}. \qquad (3)$$

Using the unitary condition $\det |M| = 1$, the transmission coefficient of the multilayer film can be written as

$$T = \frac{4}{|M|^2 + 2} = \frac{4}{m_{11}^2 + m_{12}^2 + m_{21}^2 + m_{22}^2 + 2}, \quad (4)$$

where $|M|^2$ denotes the sum of the squares of the four elements of $M$.

Now we introduce the mirror symmetry into the dielectric multilayer. It is proved that the transmission coefficient of the light wave through the multilayers with internal mirror symmetry can be expressed as[6]

$$T = \frac{4}{|M|^2 + 2} = \frac{4}{(m_{12} + m_{21})^2 + 4}. \qquad (5)$$

Obviously, under the condition

$$m_{12} + m_{21} = 0, \qquad (6)$$

perfect transmissions can be expected in the dielectric multilayer with mirror symmetry.

The above theoretical analysis can be demonstrated by experiment; for example, optical transmission measurements on the symmetric Fibonacci $SiO_2/TiO_2$ multilayer films. It is well known that the Fibonacci sequence can be produced by repeated application of the substitution rules $A \rightarrow AB$ and $B \rightarrow A$. Since Merlin *et al.*[7] first reported the realization of Fibonacci superlattices in 1985, much attention has been paid to the exotic wave phenomena of Fibonacci systems.[8] In 1987, Kohmoto *et al.*[9] proposed the photonic Fibonacci multilayers (FM's). Very recently, the experiments on the optical dielectric multilayers with Fibonacci structure were reported.[10] The symmetric Fibonacci sequence, which we reported for the first time,[6,11] can be constructed in the following way. The $j$th generation of the sequence can be expressed as $S_j = \{G_j, H_j\}$, where $G_j$ and $H_j$ are Fibonacci sequences. $G_j$ and $H_j$ obey the recursion relations

$$G_j = G_{j-2}G_{j-1},$$
$$H_j = H_{j-1}H_{j-2}, \tag{7}$$

with $G_0 = H_0 = B$ and $G_1 = H_1 = A$. Therefore

$$S_j = G_{j-2}G_{j-1}H_{j-1}H_{j-2}. \tag{8}$$

For example, the fourth sequence is $S_4 = \{BAABAABAAB\}$, which possesses a mirror symmetry. In experiments, by using electron-gun evaporation, the symmetric Fibonacci $SiO_2/TiO_2$ multilayer (SFM) films were fabricated on the glass substrate. The refractive indices are $n_B = n_{SiO_2} = 1.46$ and $n_A = n_{TiO_2} = 2.30$ around the wavelength of 700 nm. Before the evaporation, the pressure of the chamber was lower than $2 \times 10^{-5}$ Torr, and the films were formed under an oxygen atmosphere: the pressure is $2 \times 10^{-4}$ Torr for $TiO_2$ deposition and $0.8 \times 10^{-4}$ Torr for $SiO_2$. For the sake of simplification, the thickness of these two materials as chosen to satisfy the condition $n_A d_A = n_B d_B$, which gives the same phase shift in two materials, i.e. $\delta_A = \delta_B = \delta$. The central wavelength was set to 700 nm or so, and therefore $d_A = (700 \text{ nm})/4n_A \cong 76.1$ nm and $d_B = (700 \text{ nm})/4n_B \cong 120.0$ nm. And optical transmission spectra were measured by using a U-3410 spectrophotometer in the range of wavelength from 185 nm to 2600 nm.



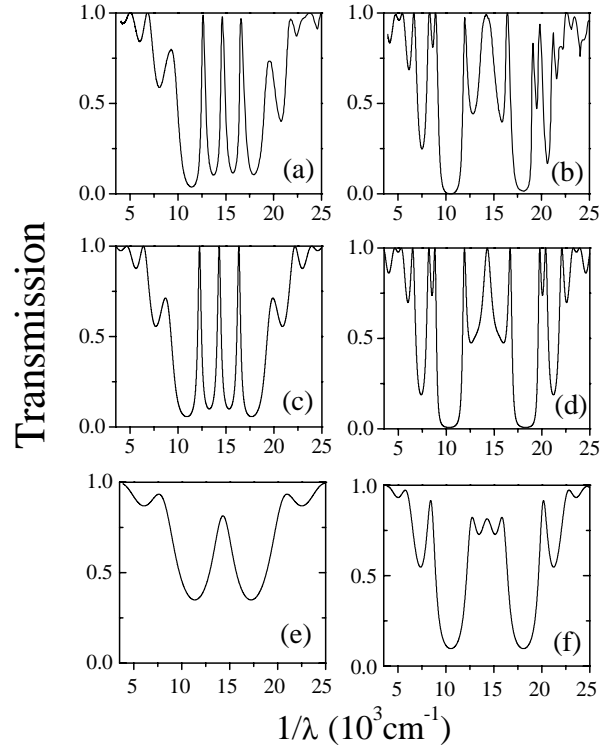Fig. 1. The measured and calculated transmission coefficient $T$ versus the wave number $\lambda^{-1}$ for the symmetrical Fibonacci $TiO_2/SiO_2$ multilayers (SFM's). The SFM $S_5$ with 16 layers, (a) measured and (c) calculated; the SFM $S_6$ with 26 layers, (b) measured and (d) calculated; respectively. The calculated transmission coefficient $T$ versus the wave number $\lambda^{-1}$ for the Fibonacci $TiO_2/SiO_2$ multilayers (FM): (e) the FM $H_5$ with 8 layers, and (f) the FM $H_6$ with 13 layers.

Figures 1(a) and 1(c) show the experimentally measured transmission coefficients as a function of wave number for the symmetric Fibonacci $SiO_2/TiO_2$ multilayer (SFM) films with generations $S_5$ and $S_6$, respectively. Obviously many perfect (or almost perfect) transmissions have been observed. The measurement transmission spectra [shown in Figs. 1(a) and 1(c)] are in good agreement with the numerical calculations [shown in Figs. 1(b) and 1(d)]. And it is quite interesting to compare the optical transmission of the SFM with that of the Fibonacci $SiO_2/TiO_2$ multilayers (FM) without symmetry. Figures 1(e) and 1(f) show the calculated transmission coefficients of $SiO_2/TiO_2$ FM films. It is evident that the poor transmission of the optical wave usually occurs in ordinary Fibonacci dielectric multilayers; while more resonant modes with perfect transmission indeed exist in symmetric Fibonacci

multilayers. The reason is that the mirror symmetry in the system may create special structures in some transfer matrix elements which make it easier to satisfy the condition of perfect transmission as shown in Eq. (6). It is well known that the dielectric multilayer films have many potential applications such as narrow-band filters and frequency selectors. In periodic multilayers, the number of tunable structural parameters is quite limited; while in the case of quasiperiodic structures, especially when the mirror symmetry is induced, the structure of transmission spectra becomes more complicated, and many perfect transmission windows can be presented.

As we have seen in the above discussion, the perfect transmission will definitely occur if the mirror symmetry is introduced into the structure. Actually, the perfect transmission can be controlled at certain wavelengths if the special structure with
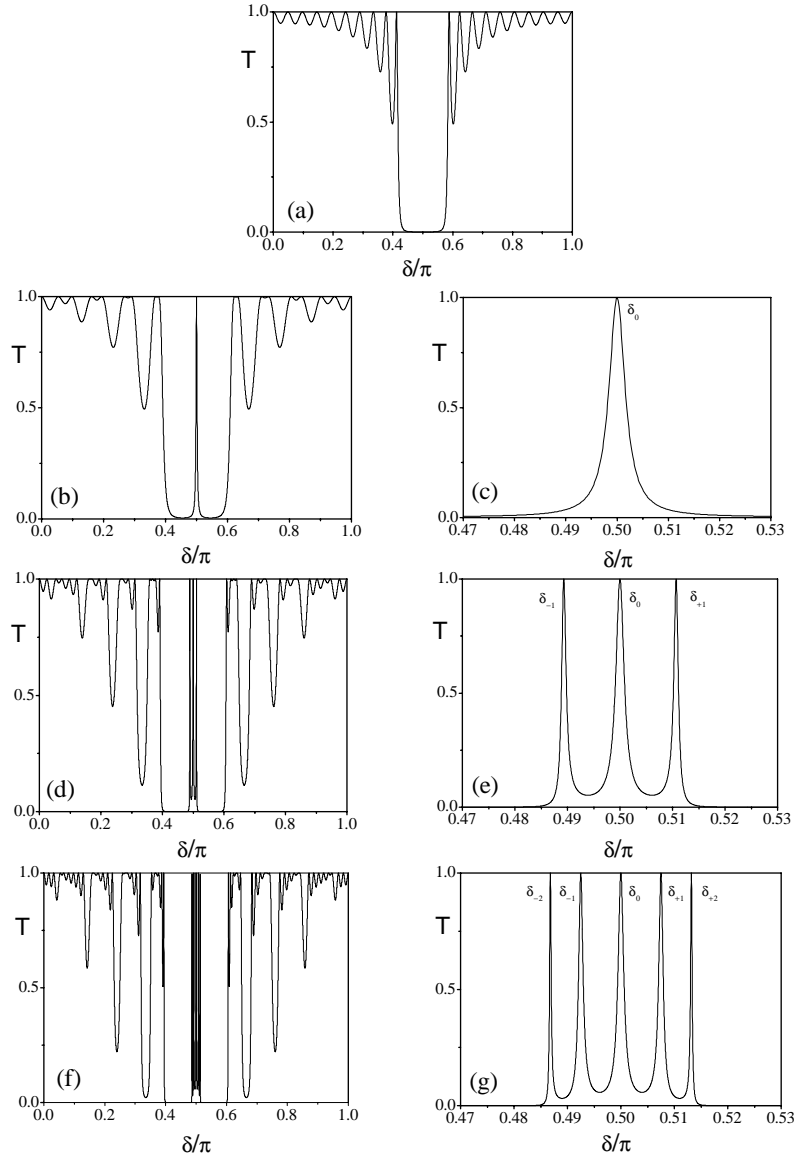


Fig. 2. The calculated transmission coefficient $T$ as a function of the optical phase $\delta$ for the symmetric multilayers with defects (SMD's): (a) The periodic multilayer as $\mathfrak{R}_1$; (b) The SDM as $\mathfrak{R}_2$; (c) The enlargement of (b) around $\delta_0 = 0.5\pi$; (d) The SDM as $\mathfrak{R}_3$; (e) The enlargement of (d) around $\delta_0 = 0.5\pi$; (f) The SDM as $\mathfrak{R}_3$; (g) the enlargement of (f) around $\delta_0 = 0.5\pi$.

symmetry is achieved. Here we present a method to obtain the resonant mode at the specified wavelength in so-called symmetric multilayers with defects (SMD's). For the simplification of calculations, we still choose two kinds of dielectric materials, $A$ and $B$, with the thicknesses of two different layers set to satisfy $n_A d_A = n_B d_B$, and therefore with the optical phases corresponding to the two different layers being the same, $\delta_A = \delta_B = \delta$. Firstly, it is well known[12] that there is a wide gap around $\delta = 0.5\pi$ [as shown in Fig. 2(a)] in the optical transmission spectrum of the periodic multilayer, which has ten periods and is composed by the layers $\mathfrak{R}_1 = \{ABABABABAB\}$. In the second, if the multilayer film is constructed as $\mathfrak{R}_2 = \mathfrak{R}_1 \cup \mathfrak{R}_1^{-1} = \{ABABABABABBABABABABA\}$ (where $\mathfrak{R}_0^{-1}$ is defined as $\mathfrak{R}_1^{-1} = \{BABABABABA\}$), there will be one perfect transmission peak at the optical phase of $\delta_0 = 0.5\pi$ within the central gap [shown in Figs. 2(b) and 2(c)]. Obviously, this perfect transmission comes from the defect mode. Thirdly, if we construct the symmetric multilayer with defects (SMD) as $\mathfrak{R}_3 = \mathfrak{R}_2 \cup \mathfrak{R}_2$, three perfect transmission peaks occur in the central gap, as shown in Figs. 2(d) and 2(e). The optical phases of these three resonant modes are distributed at $\delta_{-1}$, $\delta_0$ and $\delta_1$, where $\Delta\delta = \delta_1 - \delta_0 = \delta_0 - \delta_{-1}$. Furthermore, if the fourth SMD is formed as $\mathfrak{R}_4 - \mathfrak{R}_3 \cup \mathfrak{R}_2 = \mathfrak{R}_2 \cup \mathfrak{R}_2 \cup \mathfrak{R}_2$, as can be seen in Figs. 2(f) and 2(g), five perfect transmission peaks occur at the central gap of the transmission spectrum. They locate at the optical phases $\delta_{-2}$, $\delta_{-1}$, $\delta_0$, $\delta_1$ and $\delta_2$, respectively. Therefore, after $k$ operations, the SMD possesses $k - 1$ units of $\mathfrak{R}_2$, and there will be $2k - 3$ perfect peaks around $\delta_0$ in the corresponding optical transmission spectrum, i.e. $2k - 3$ resonant modes can be achieved at the specified phases. And if the two nearest peaks of perfect transmissions away from the central peak at $\delta_0$ are marked by "$-1$" and "$+1$," respectively, the corresponding phases $\delta_{-1}$ or $\delta_1$ approximately satisfy the relation $\Delta\delta = \delta_1 - \delta_0 = \delta_0 - \delta_{-1} \cong 0.02\,\pi\exp(-k.2.35)$, as shown in Fig. 3, where $k - 1$ is the number of $\mathfrak{R}_2$ in the SMD constructed by the above operation. Once the dielectric materials and their thicknesses in the multilayers are chosen, the phase in the optical transmission spectrum is directly related to the wavelength of the light. Therefore, it seems that the perfect transmissions can be controlled at certain
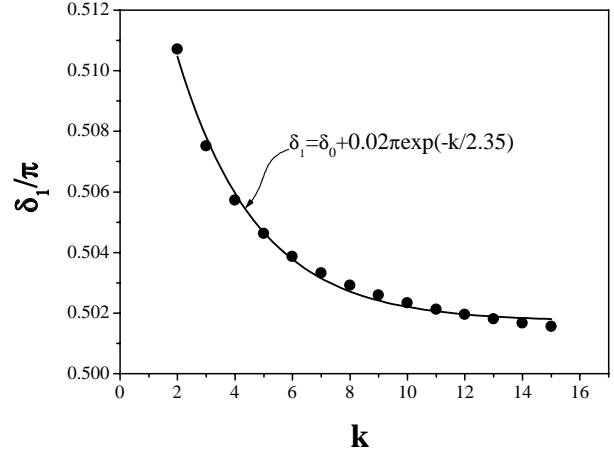


Fig. 3. The relation between the optical phase $\delta_1$ of the nearest perfect peaks away from the central peak at $\delta_0 = 0.5\pi$ and the number of $\mathfrak{R}_2$, i.e. $k$, in the SMD described in the text.

wavelengths following this way. The resonant modes, which are required in the optoelectronic devices, can be achieved by designing the dielectric multilayers with a specific symmetric structure.

To summarize, based on the transfer-matrix method, we have investigated the propagation of electromagnetic waves in the dielectric multilayers with symmetric structures. It is shown that if the mirror symmetry is induced to the structure, the resonant transmission will definitely occur. Experimental measurements in symmetric Fibonacci $SiO_2/TiO_2$ multilayer films are in good agreement with the theoretical predictions. We also have presented the way to control the perfect transmission taking place at certain wavelengths. It can be expected that this finding will contribute to the multiwavelength narrow band optical filters, the wavelength division multiplexing (WDM) system and photonic integrated circuits, where high-transmission and high-resolution monofrequency outputs are particularly desired.

## Acknowledgments

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