EUROPHYSICS LETTERS

OFFPRINT

Vol. 63 • Number 6 • pp. 853–859

Resonant transmission and frequency trifurcation of light waves in Thue-Morse dielectric multilayers

* * *

F. Qiu, R. W. Peng, X. Q. Huang, Y. M. Liu, M. Wang, A. Hu and S. S. Jiang

Published under the scientific responsability of the

EUROPEAN PHYSICAL SOCIETY

Incorporating

JOURNAL DE PHYSIQUE LETTRES • LETTERE AL NUOVO CIMENTO

EUROPHYSICS LETTERS 15 September 2003

Europhys. Lett., **63** (6), pp. 853–859 (2003)

Resonant transmission and frequency trifurcation of light waves in Thue-Morse dielectric multilayers

F. Qiu, R. W. Peng(*), X. Q. Huang, Y. M. Liu, M. Wang,
A. Hu and S. S. Jiang

National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University - Nanjing 210093, PRC

(received 3 April 2003; accepted in final form 11 July 2003)

PACS. 68.60.-p – Physical properties of thin films, nonelectronic.
PACS. 61.44.Br – Quasicrystals.
PACS. 78.67.Pt – Multilayers; superlattices.

Abstract. – We present in this letter the observation of the optical resonant transmission of Thue-Morse (TM) dielectric multilayers. For the first time the frequency trifurcation feature has been experimentally demonstrated. This effect can be analogous to the electronic energy spectrum of a TM system, which has not yet been directly obtained in experiments. The resonant transmission originates from the positional correlation of the basic units in the TM system, and is sensitive to the modulation of optical thickness. The experimental results are in good agreement with the theoretical analysis.

Recently, the photonic localization in dielectric structures has attracted much attention [1], which can be considered as a counterpart of electronic behavior in solids. Strong localization of photons may lead to a photonic band gap (PBG) in the electromagnetic wave spectrum. A complete PBG in this type of material may have potential applications in photonic information technology [2]. Up to now, most studies on PBG material are concentrated on periodic structures. Yet localization is a common effect in aperiodic structures [3–5]. In the last decade, the notion of aperiodic order has been intensively investigated, which on the one hand stimulated much interest in one-dimensional systems [6, 7]. On the other hand, it activated the desires of potential technological applications [8]. To deal with the propagation of electromagnetic wave in an aperiodic layered structure, Kohmoto *et al.* proposed the photonic Fibonacci multilayers in 1987 [3]. Later on, the optical properties of Fibonacci and random multilayers were compared numerically [4]. However, optical Fibonacci dielectric multilayers were not experimentally achieved until 1994 [9]. Recently, other interesting issues of the optical propagations have also been investigated in quite a few quasiperiodic structures [10, 11].

One of the well-known examples of 1D aperiodic systems is the Thue-Morse (TM) sequence [4]. The TM lattice is not quasiperiodic but deterministically aperiodic, which can be obtained by the inflation rules: $A \to AB$ and $B \to BA$. It has been reported theoretically that the TM lattice has a singular continuous Fourier spectrum [12] and a Cantor-like phonon

(*) E-mail: rwpeng@nju.edu.cn

property [13]. Some other important properties, such as electronic spectra, optical transmissions and spin excitations in the TM structures, have also been studied theoretically [14–16]. In particular, Liu [15] has numerically investigated the localization property of light in a TM dielectric multilayer by introducing a localization index. He also theoretically predicted the effect of modulations of both the refractive index and the phase shift in this system. In this letter, we present an experimental observation on the resonant transmission of light wave in TM dielectric multilayers. It is shown that the resonant transmission originates from a special positional correlation between two kinds of blocks (A and B) in the structure. The optical trifurcation feature has been experimentally demonstrated for the first time. We also find that the resonant transmission around the central wavelength is very sensitive to the modulation of the optical thickness. The experimental results are in good agreement with the theoretical analysis based on the method of the transfer matrix.

According to the inflation rules: $A \to AB$ and $B \to BA$, it is ready to obtain the generations S_n of the TM sequence as follows:

$$\begin{aligned} S_0 &= [A], \\ S_1 &= [AB], \\ S_2 &= [ABBA], \\ S_3 &= [ABBABAAB], \\ S_4 &= [ABBABAABBAABABBA], \\ &\ldots \end{aligned}$$

Now we consider the optical propagation through a TM multilayer consisting of two kinds of dielectric materials A and B, with refraction indices n_A and n_B and thicknesses d_A and d_B, respectively. We use the transfer matrix method, and follow the description of the electric field in the report of Kohmoto *et al.* [3]. For normal incidence and polarization parallel to the multilayers surface, the matrix T_{AB} (or T_{BA}) represents light propagation across the interface $B \to A$ (or $A \to B$):

$$T_{AB} = T_{BA}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & n_B/n_A \end{pmatrix}. \tag{1}$$

The propagation through the layer A (or B) can be described by T_A (or T_B) as

$$T_{A(B)} = \begin{pmatrix} \cos\delta_{A(B)} & -\sin\delta_{A(B)} \\ \sin\delta_{A(B)} & \cos\delta_{A(B)} \end{pmatrix}, \tag{2}$$

where the phase is given by $\delta_A = n_A d_A \kappa$ (or $\delta_B = n_B d_B \kappa$). Here the wave vector $\kappa = 2\pi/\lambda$. Therefore, the product matrix

$$M_n = \begin{pmatrix} m_{11}(n) & m_{12}(n) \\ m_{21}(n) & m_{22}(n) \end{pmatrix} \tag{3}$$

is related to the incoming wave, the reflected wave and the transmitted wave. The transmission coefficient of the light through a layered medium can be expressed as

$$T_n = \frac{4}{\sum_{i,j=1}^{2} m_{ij}^2(n) + 2}. \tag{4}$$

In the case of the Thue-Morse dielectric multilayers, it can be found that the even-order TM multilayer (ETM) has the characteristic of mirror symmetry. It follows that the diagonal elements of M_n should satisfy

$$m_{11}(n) = m_{22}(n) \quad (\text{for even } n,\ i.e.,\ \text{ETM}). \tag{5}$$

Similarly, for an odd-generation TM multilayer (OTM), the off-diagonal elements of M_n should obey

$$m_{12}(n) = -m_{21}(n) \quad (\text{for odd } n,\ i.e.,\ \text{OTM}). \tag{6}$$

Considering M_n is a unimodular matrix, *i.e.*, $\det|M_n| = 1$, it can be derived that for the TM dielectric multilayer, the transmission coefficient T_n of eq. (4) is determined only by two elements, *i.e.*,

$$T_n = \begin{cases} \dfrac{4}{\left[m_{12}(n) + m_{21}(n)\right]^2 + 4} & (\text{for even } n,\ i.e.,\ \text{ETM}), \\ [12pt] \dfrac{4}{\left[m_{11}(n) - m_{22}(n)\right]^2 + 4} & (\text{for odd } n,\ i.e.,\ \text{OTM}). \end{cases} \tag{7}$$

Obviously, if the conditions

$$\begin{cases} m_{12}(n) + m_{21}(n) = 0 & (\text{for even } n,\ i.e.,\ \text{ETM}), \\ m_{11}(n) - m_{22}(n) = 0 & (\text{for odd } n,\ i.e.,\ \text{OTM}) \end{cases} \tag{8}$$

are satisfied, a perfect transmission of light will definitely occur in the TM dielectric multilayers.

In the experiments, silicon dioxide (SiO_2) and titanium dioxide (TiO_2) were chosen as dielectric materials A and B, respectively. Their refractive indices are $n_A = 1.47$ and $n_B = 2.30$ around the wavelength of 700 nm. The multilayers were fabricated on the substrate of $K9$ glass by the electron-gun evaporation (SHOWA SGC-12SA). The substrate temperature was 300 °C. The films were deposited in an oxygen atmosphere with the pressure 2×10^{-4} torr for TiO_2 and 0.8×10^{-4} torr for SiO_2. The deposition velocities of SiO_2 and TiO_2 were about 0.6 nm/s and 0.1 nm/s, respectively. The thickness of the films was controlled by the quartz-crystal monitoring with the frequency of 5 MHz, and in the meantime assisted by the optical monitoring. For the sake of simplification, the thicknesses of the two materials were chosen to satisfy $n_A d_A = n_B d_B$. In this case, the phase shifts through these two thin films are the same, *i.e.* $\delta_A = \delta_B = \delta$. The optical thickness of each layer is around a quarter wavelength, $\lambda_0/4$ (the central wavelength is $\lambda_0 = 700$ nm). These conditions imply that the physical thicknesses of two materials are given by $d_A = (700\,\text{nm})/(4 \times n_A) = 119.0$ nm and $d_B = (700\,\text{nm})/(4 \times n_B) = 76.1$ nm, respectively.

The optical transmission of the TM SiO_2/TiO_2 multilayer films has been studied. The transmission spectra were measured by the spectrophotometer (HITACHI U-3410), and the wavelength was tuned from 400 nm to 2500 nm. Figure 1 illustrates the calculated and the measured transmission coefficients as a function of wave number for the TM films with the generations S_3, S_4, and S_5, respectively. It can be seen that with the increasing of the number of layers, the aperiodicity in the structure becomes more effective, and more and more transmission zones die out gradually and a 1D photonic band gap will eventually come out. This feature is very similar to the optical behavior in quasiperiodic structures [4, 9, 10]. Meanwhile, by increasing the number of layers in the TM multilayer, more and more perfect transmission peaks indeed appear. The observed transmissions are in good agreement with the numerical calculations. (Comparing with fig. 1(e), the transmission peaks shown in fig. 1(f)

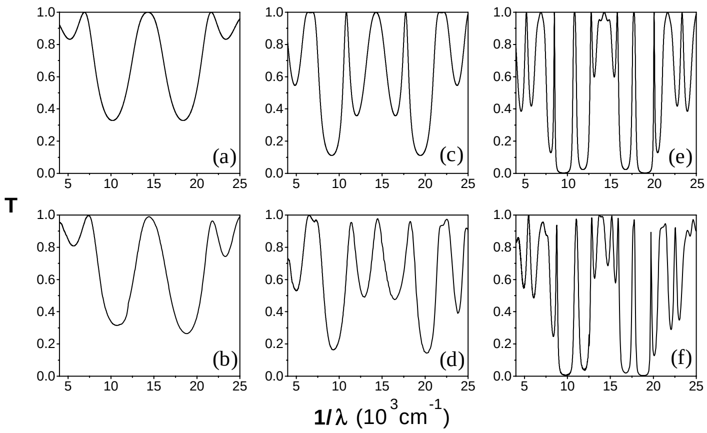

Fig. 1 – Calculated and measured transmission coefficients *vs.* wave number for the Thue-Morse (TM) SiO_2/TiO_2 multilayer films with different generations. TM S_3 with 8 layers: (a) calculated and (b) measured; TM S_4 with 16 layers: (c) calculated and (d) measured; TM S_5 with 32 layers: (e) calculated and (f) measured.

have a slight shift around central wavelength, which mainly comes from the deviation of optical thicknesses between SiO_2 and TiO_2. This deviation is below 5%.)

Now we focus on the completely transparent states, *i.e.* the perfect transmission peaks around the central wavelength. Figure 2 shows the frequency distribution of the complete transmission peaks in different generations of the TM structures. The resonant transmission peaks for each generation can be considered as a band (as shown in fig. 2), thereafter, each band consists of three subbands. In other words, the frequency of resonant transmissions in the TM multilayer possesses a trifurcation feature. For example, the central subband of the 6th generation (part O) is the same as the band of 5th generation, while each of the other subbands (part S) is similar to the band of 4th generation. Due to the self-similarity of the

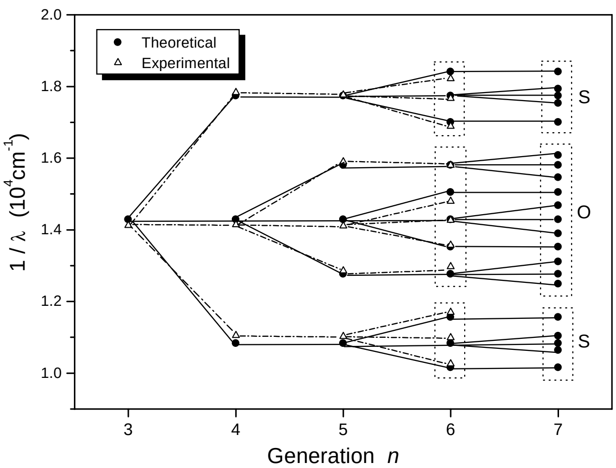

Fig. 2 – The frequency distribution of the complete transmission states in the TM SiO_2/TiO_2 multilayer films for different generations S_n, where n is the number of generations of TM structure.

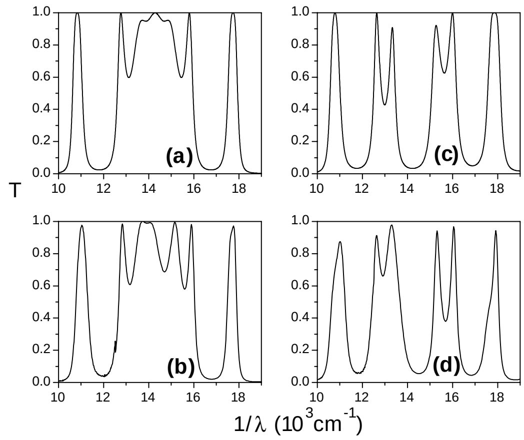

Fig. 3 – The transmission spectrum of the TM SiO_2/TiO_2 multilayer films (S_5 with 32 layers) around the central wavelength $\lambda_0 = 700\,\mathrm{nm}$. In the case of $x = y = 0.25$: (a) calculated and (b) measured. In the case of $x = 0.3$, $y = 0.2$: (c) calculated and (d) measured, where x (or y) describes the deviation of the optical thickness to the central wavelength for the material SiO_2 (or TiO_2) as defined in the text.

spectrum, we can count the number of the completely transparent states in the TM multilayer with any n-th generation. Defining m_n as the number of the mode of the resonant transmission (or the number of the completely transparent states) around the central wavelength, we have

$$m_{n+2} = m_n + m_{n+1} + m_n \quad (n \geq 3), \tag{9}$$

with the initial conditions $m_3 = 1$ and $m_4 = 3$. Finally, we get

$$m_n = 2m_{n-1} \pm 1 = \frac{2^{n-1} \pm 1}{3}, \quad \text{where} \begin{cases} + \text{ for even } n, \ i.e., \ \text{ETM}, \\ - \text{ for odd } n, \ i.e., \ \text{OTM}. \end{cases} \tag{10}$$

According to eq. (10), the mode number of resonant transmission is indeed related to the inner feature of the TM structure. Actually, the resonant transmissions should originate from a special positional correlation between two kinds of blocks (A and B) in the TM structure. As a matter of fact, trifurcation in the electronic energy spectrum of a TM superlattice has been theoretically predicted by Qin *et al.* [14], but it has not yet been experimentally observed. In some senses, the photonic spectrum can be regarded as an analogue to the electronic spectrum. So it is worthwhile to measure experimentally the optical trifurcation of resonant transmissions in TM dielectric multilayers.

In order to demonstrate that resonant transmission of light is related to the positional correlation in the TM structure, we investigate the effect of the modulation of the optical thickness on the resonant transmission in a TM multilayer. In the above discussions we always set $n_A d_A = n_B d_B$, which means that the phase shift in each layer is identical. Now we consider the scenario $n_A d_A \neq n_B d_B$, meaning that the optical thickness, and hence the phase shift is aperiodically modulated. To some extent, the optical thickness modulation may be considered as positional modulation [15]. Define the optical thicknesses of the two materials as

$$\begin{aligned} D_A &= n_A d_A = x\lambda_0, \\ D_B &= n_B d_B = y\lambda_0, \end{aligned} \tag{11}$$

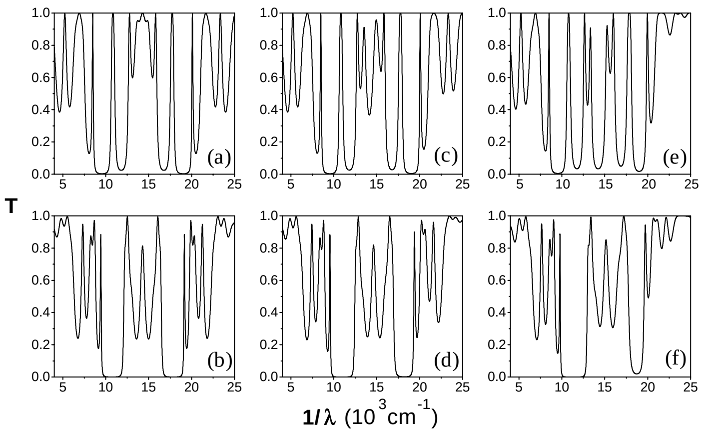

Fig. 4 – The calculated optical transmission spectra of TM (S_5 with 32 layers) and Fibonacci (S_8 with 34 layers) SiO_2/TiO_2 multilayer (ML) films with different optical thickness modulation, respectively. In the case of $x = y = 0.25$: (a) Thue-Morse ML and (b) Fibonacci ML. In the case of $x = 0.27$, $y = 0.23$: (c) Thue-Morse ML and (d) Fibonacci ML. In the case of $x = 0.3$, $y = 0.2$: (e) Thue-Morse ML and (f) Fibonacci ML, where x (or y) describes the deviation of the optical thickness to the central wavelength for the material SiO_2 (or TiO_2) as defined in the text.

where λ_0 is the central wavelength and x (or y) stands for the deviation of the optical thickness to the central wavelength for the material A (or B). Then the corresponding phase shifts can be expressed as $\delta_A = 2\pi D_A/\lambda = x2\pi\lambda_0/\lambda$, and $\delta_B = 2\pi D_B/\lambda = y2\pi\lambda_0/\lambda$, respectively. Obviously, if $x = y = 0.25$, we are back to the quarter-wavelength case with the same phase shift for each layer. Figure 3 shows the experimental and calculated transmission spectra of the TM SiO_2/TiO_2 multilayer films around the previous central wavelength λ_0 in two different cases: (a) $x = y = 0.25$ (*i.e.*, without optical thickness modulation) and (b) $x = 0.3$ and $y = 0.2$ (*i.e.*, with optical thickness modulation). It is obvious that when the optical thickness modulation is introduced, around the central wavelength λ_0, the former completely transparent region (shown in fig. 3(a) and (b)) becomes an almost completely reflected zone (shown in fig. 3(c) and (d)), while in the region far from the central wavelength λ_0 the transmission spectrum has not been significantly influenced. This situation does not occur to a Fibonacci multilayer. Figure 4 provides the calculated optical transmission spectra of TM and Fibonacci SiO_2/TiO_2 multilayer films with different optical thickness modulation, respectively. It can be seen that in a Fibonacci dielectric multilayer the optical transmission only has a gradual shift for the same deviation of the optical thickness. We therefore conclude that the resonant transmission of light in a TM multilayer is sensitive to the optical thickness modulation especially around the central frequency.

In summary, in this letter the resonant transmission and its frequency trifurcation features have been investigated in the Thue-Morse (TM) dielectric multilayers. We select SiO_2/TiO_2 as the building blocks to demonstrate the optical properties of such a structure. It is found that the completely transparent states are related to the special positional correlation in the TM structure, and are sensitive to the optical thickness modulation around the central wavelength. We suggest that the optical behaviors illustrated in this letter could be helpful to understand the electronic counterparts in the TM system, and it may also have potential applications in optical communication.

* * *

This work was supported by the grants from National Nature Science Foundation of China (19904006, 10021001, 90201039, 10274029), the State Key Program for Basic Research from the Ministry of Science and Technology of China (C01CB610620, G1998061410), NSF of Jiangsu (BK2002408) and partly by Fok Ying Tung Education Foundation.

REFERENCES

[1] John S., *Phys. Rev. Lett.*, **58** (1987) 2486; Yablonovitch E., *Phys. Rev. Lett.*, **58** (1987) 2059; Yablonovitch E., Gmitter T. J. and Bhat R., *Phys. Rev. Lett.*, **63** (1987) 1950.

[2] Joannopoulos J. D., Villeneuve P. R. and Fan S., *Nature (London)*, **386** (1997) 143.

[3] Kohmoto M., Sutherland B. and Iguchi K., *Phys. Rev. Lett.*, **58** (1987) 2436.

[4] Riklund R. and Severin M., *J. Phys. C*, **21** (1988) 3217; Dulea M., Severin M. and Riklund R., *Phys. Rev. B*, **42** (1990) 3680.

[5] Chan Y. S., Chan C. T. and Liu Z. Y., *Phys. Rev. Lett.*, **80** (1998) 956; Zoorob M. E., Charlton M. D. B., Parker G. J., Baumberg J. J. and Netti M. C., *Nature (London)*, **404** (2000) 740; Kaliteevsk M. A., Brand S., Abram R. A., Krauss T. F., Dela Rue R. and Millar P., *Nanotechnology*, **11** (2000) 274.

[6] Kohmoto M., Kadanoff L. P. and Tang C., *Phys. Rev. Lett.*, **50** (1983) 1870; Ostlund S. and Pandit R., *Phys. Rev. B*, **29** (1984) 1394; Levin D. and Steinhardt L. P., *Phys. Rev. Lett.*, **53** (1984) 2477; Albuquerque E. L. and Cottam M. G., *Phys. Rep.*, **376** (2003) 225.

[7] Merlin R., Bajema K., Clarke R., Juang F. Y. and Bhattacharya P. K., *Phys. Rev. Lett.*, **55** (1985) 1768; Todd J., Merlin R., Clarke R., Monhanty K. M. and Axe J. D., *Phys. Rev. Lett.*, **57** (1986) 1157.

[8] Peng R. W., Hu A. and Jiang S. S., *Appl. Phys. Lett.*, **59** (1991) 2512; Macia E., *Appl. Phys. Lett.*, **73** (1998) 3330; Peng R. W. *et al.*, *Appl. Phys. Lett.*, **80** (2002) 3063.

[9] Gellermann W., Kohmoto M., Sutherland B. and Taylor P. C., *Phys. Rev. Lett.*, **72** (1994) 633.

[10] Chow L. and Guenther K. H., *J. Opt. Soc. Am. A*, **10** (1993) 2231; Huang X. Q., Liu Y. Y. and Do M., *Solid State Commun.*, **87** (1993) 601; Hattori T., Tsurumachi N., Kawato S. and Nakatsuka H., *Phys. Rev. B*, **50** (1994) 4220.

[11] Peng R. W., Wang M., Hu A., Jiang S. S., Jin G. J. and Feng D., *Phys. Rev. B*, **57** (1998) 1544; Yang X. B., Liu Y. Y. and Fu X. J., *Phys. Rev. B*, **59** (1999) 4545; Huang X. Q., Jiang S. S., Peng R. W. and Hu A., *Phys. Rev. B*, **63** (2001) 245104; Dal Negro L., Oton C. J., Gaburro Z., Pavesi L., Johnson P., Lagendijk A., Righini R., Colocci M. and Wiersma D. S., *Phys. Rev. Lett.*, **90** (2003) 055501.

[12] Queffelec M., *Substitution Dynamical Systems-Spectral Analysis, Lect. Notes Math.*, Vol. **1294** (Spinger, Berlin) 1987; Godreche C. and Luck J. M., *J. Phys. A*, **23** (1990) 3769.

[13] Axel F. and Peyriere J., *J. Stat. Phys.*, **57** (1989) 1013.

[14] Qin M. G., Ma H. R. and Tsai C. H., *J. Phys. Condens. Matter*, **2** (1990) 1059; Chakrabarti A., Karmakar S. N. and Moitra R. K., *Phys. Rev. Lett.*, **74** (1995) 1403; Anton Bovier and Jean-Michel Ghez, *J. Phys. A*, **28** (1995) 2313.

[15] Liu N. H., *Phys. Rev. B*, **55** (1997) 3543.

[16] Chattopadhyay S. and Chakrabarti A., *J. Phys. Condens. Matter*, **12** (2000) 5681.