EUROPHYSICS LETTERS 1 December 2004

Europhys. Lett., **68** (5), pp. 658–663 (2004)
DOI: `10.1209/epl/i2004-10261-y`

Omnidirectional reflection of electromagnetic waves on Thue-Morse dielectric multilayers

F. Qiu, R. W. Peng(*), X. Q. Huang, X. F. Hu, Mu Wang, A. Hu,
S. S. Jiang and D. Feng

National Laboratory of Solid State Microstructures and
Department of Physics, Nanjing University - Nanjing 210093, PRC

received 6 August 2004; accepted in final form 1 October 2004

PACS. `61.44.Br` – Quasicrystals.
PACS. `42.70.Qs` – Photonic bandgap materials.
PACS. `78.67.Pt` – Multilayers; superlattices.

Abstract. – We report here the reflection of electromagnetic waves in self-similar Thue-Morse dielectric multilayers, which presents the features of multiple omnidirectional photonic bandgaps (PBGs). The number and the width of omnidirectional PBG depend on the ratio of the refraction indexes and the thicknesses of the dielectric materials. The theoretical result is partly verified by optical observation in Thue-Morse TiO_2/SiO_2 multilayers with visible and near-infrared light. Our investigations provide a new approach to achieve the omnidirectional reflection in multiple frequency ranges. With this progress, the application of dielectric reflection mirrors can be further widened.

In recent years, much attention has been paid to photonic crystals [1], which forbids the propagation of photons with a certain range of energies known as the photonic bandgaps (PBGs). The engineering of the PBG offers potential applications in optoelectronics and optical communication [2–5]. It is known that three-dimensional (3D) photonic crystals can achieve a complete PBG. Unfortunately, there are still two formidable difficulties in applying 3D photonic crystals as devices: One is making 3D periodic dielectric structures with a feature size comparable to the wavelength of visible light; the other is achieving dielectric contrasts to obtain a forbidden gap that overlaps in all directions within the Brillouin zone. Compared with the 3D case, one-dimensional (1D) photonic crystals, *i.e.*, dielectric multilayers, are much easier to fabricate. But for a long time, it had been thought that it was impossible for 1D photonic crystals to achieve a PBG for different polarization and various incident angles. Fortunately, in 1998, Yink *et al.* [6] recognized and demonstrated that under proper conditions, one-dimensional (1D) periodic dielectric structures can reflect light from all incident angles and any polarization, and finally make an omnidirectional PBG. Since then, great interest has been paid to omnidirectional dielectric mirrors based on the 1D dielectric multilayers [7]. However, up to now most works focus on the periodic structure and only one PBG exists

---

(*) E-mail: `rwpeng@nju.edu.cn`

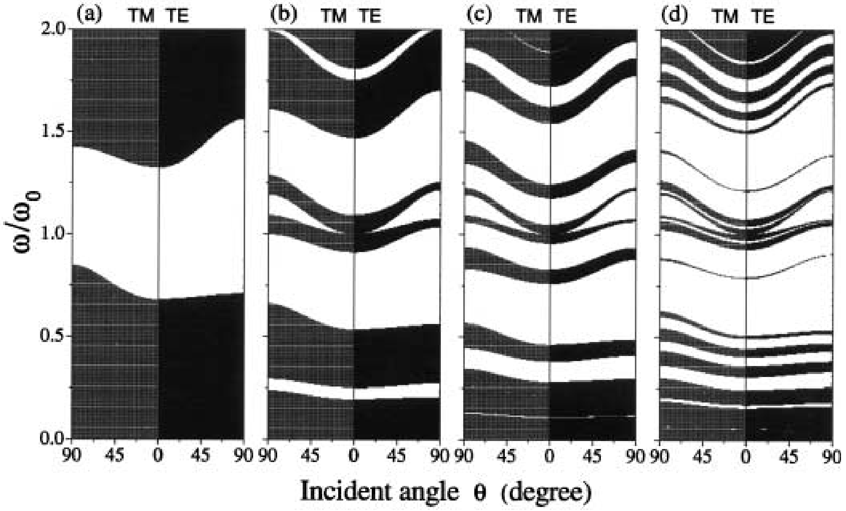

Fig. 1 – The calculated photonic band structures of the periodic and the Thue-Morse dielectric multilayers in terms of frequency ω and incident angle θ. Each multilayer consists of two building blocks A, B with $n_A = 1.6$ and $n_B = 4.6$, thicknesses $d_A = 1.25a/n_A$ and $d_B = 1.25a/n_B$, respectively (where a is constant). (a) The periodic multilayer with 8 layers. The Thue-Morse multilayer S_n: (b) S_3 with 8 layers; (c) S_4 with 16 layers; (d) S_5 with 32 layers. Note that, in each figure, the left part is for the transverse magnetic (TM) mode, while the right part for the transverse electric (TE) mode. And the dark (or gray) area stands for allowed photonic bands, while the white area for forbidden bands.

in a period of the reciprocal space. In this letter, we report omnidirectional PBGs in Thue-Morse dielectric multilayers. Due to the self-similarity in the internal structure, multiple omnidirectional PBGs exist in this non-periodic dielectric microstructure. Application such as multichannel optical devices may benefit from this observation.

The Thue-Morse sequence is one of the well-known examples in 1D aperiodic structure [8]. It contains two building blocks A and B and can be produced by repeating application of the substitution rules $A \rightarrow AB$ and $B \rightarrow BA$. For example, the first few generations S_n of Thue-Morse sequence are as follows: $S_0 = \{A\}$, $S_1 = \{AB\}$, $S_2 = \{ABBA\}$, $S_3 = \{ABBABAAB\}$ and so on. The singular-continuous Fourier spectra, electronic spectra, and spin excitations in the Thue-Morse structures have been studied theoretically [9]. Very recently, the optical resonant transmission has been found both theoretically and experimentally [10]. Here, we consider the reflectivity of electromagnetic waves through a Thue-Morse dielectric multilayer for both transverse electric (TE) and transverse magnetic (TM) polarizations and for different incident angles. The Thue-Morse dielectric multilayer consists of two building blocks A and B with refraction indexes n_A and n_B, thicknesses d_A and d_B, respectively. In a multilayer, A and B are arranged according to the Thue-Morse sequence. The electromagnetic wave is incident from air to the top surface of the multilayer. With the transfer matrix method [11], the photonic band structure and the optical reflectivity through the multilayers can be numerically calculated.

Figure 1 illustrates the calculated photonic band structures of several Thue-Morse dielectric multilayers with different generations S_n ($n = 3, 4, 5$, respectively), where the building blocks have the refraction indexes $n_A = 1.6$ and $n_B = 4.6$, thicknesses $d_A = 1.25a/n_A$ and $d_B = 1.25a/n_B$, respectively, where a is the constant related to the central frequency of photons.

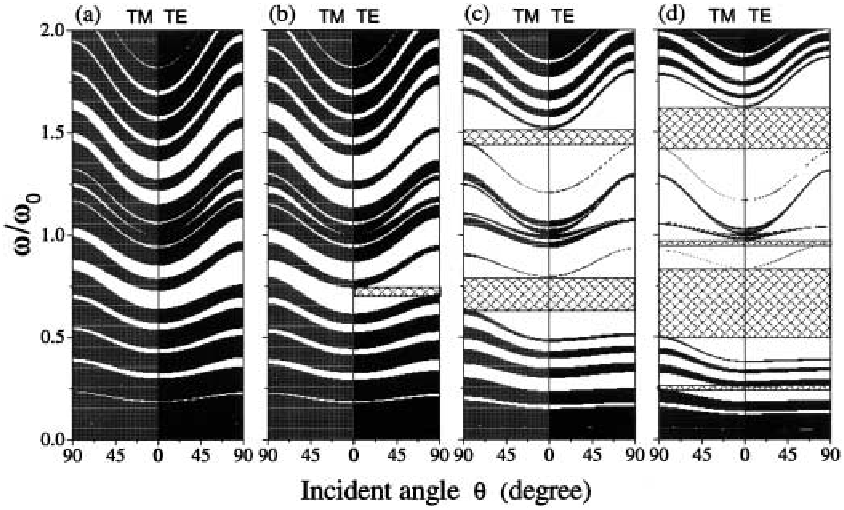

Fig. 2 – The calculated photonic band structures of the Thue-Morse dielectric multilayers S_5 with $n_A = 1.47$ and different ratio of the refraction indexes of the dielectric materials (A, B), *i.e.*, $r = n_B/n_A$. (a) $r = 1.4$; (b) $r = 1.6$; (c) $r = 3.2$; and (d) $r = 6.0$. The omnidirectional PBG is marked as a gridding square.

Different from the only PBG in the periodic dielectric multilayer (shown in fig. 1(a)), multiple PBGs can be found at the same frequency range in Thue-Morse structures (shown in figs. 1(b)-(d)). For example, the photonic band structure of the Thue-Morse multilayer with S_3 is correspondingly distributed along the central frequency $\omega_0 = 0.2 \times \frac{2\pi c}{a}$ (shown in fig. 1(b)). There are totally four PBGs in S_3 for both TE and TM polarizations: half of them are located in the high-frequency region ($\omega > \omega_0$) and others are in the low-frequency region ($\omega < \omega_0$). The PBG is flatter in the low-frequency region than that in the high-frequency one. By increasing the number of layers in the structures, more PBGs gradually emerge (shown in figs. 1(b)-(d)). Actually, due to the self-similarity of the Thue-Morse structure, the resonant transmissions around the central frequency ω_0 possess a trifurcation feature [10]. Thereafter, in the Thue-Morse multilayer S_n, the number of PBGs around ω_0, *i.e.*, g_n $(n > 3)$, can be counted as

$$g_n = 2g_{n-1} - 1 \pm 1 = 1 + \frac{2^{n-1} \pm 1}{3}, \quad \text{where} \begin{cases} + & \text{for even } n, \\ - & \text{for odd } n. \end{cases} \tag{1}$$

From this point of view, the PBGs and their number in the Thue-Morse dielectric multilayer are indeed related to the self-similarity in the structure.

According to figs. 1(b)-(d), there are indeed multiple PBGs in the Thue-Morse dielectric multilayer, but not all of them are omnidirectional PBGs, in which the light reflects regardless of the incident angle and the polarization. Actually, the width of omnidirectional PBG in the Thue-Morse structure depends on the materials and structural parameters of the building blocks A and B. Figure 2 gives the calculated photonic band structures of several Thue-Morse dielectric multilayers S_5 with different ratio $r = n_B/n_A$ of the refraction indexes of two dielectric materials, where $n_A = 1.47$. When r is small, there is not an omnidirectional PBG (as shown in fig. 2(a)). With increasing r, there are omnidirectional PBGs for the TE mode, but not for the TM mode (as shown in fig. 2(b)). If r is large enough, omnidirectional PBGs come

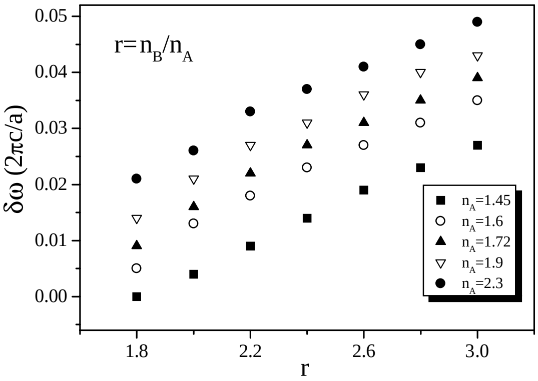

Fig. 3 – The r-dependent width of the largest omnidirectional PBG of the Thue-Morse multilayers in the low-frequency region. The ratio of the refraction indexes of two materials is $r = n_B/n_A$.

out for both TE and TM polarizations (as shown in figs. 2(c) and (d)). As in the periodic case, the width of PBGs for the TE mode is larger than the corresponding case for the TM mode in the Thue-Morse dielectric structure. Further, the width of the omnidirectional PBG is exactly determined by the refractive index and the thickness of the dielectric materials. Figure 3 shows the r-dependent width of the largest omnidirectional PBG of the Thue-Morse multilayers in the low-frequency region. It is found that, if n_A is kept the same, the width of PBG increases by increasing $r = n_B/n_A$. While for a certain value of r, if n_A enlarges, the photonic band becomes flatter, thereafter, the width of the PBG becomes larger. To some extent, fig. 3 describes how to obtain the largest omnidirectional PBG in Thue-Morse structures.

To verify the theoretical analysis, we carry out the following experiment. Silicon dioxide

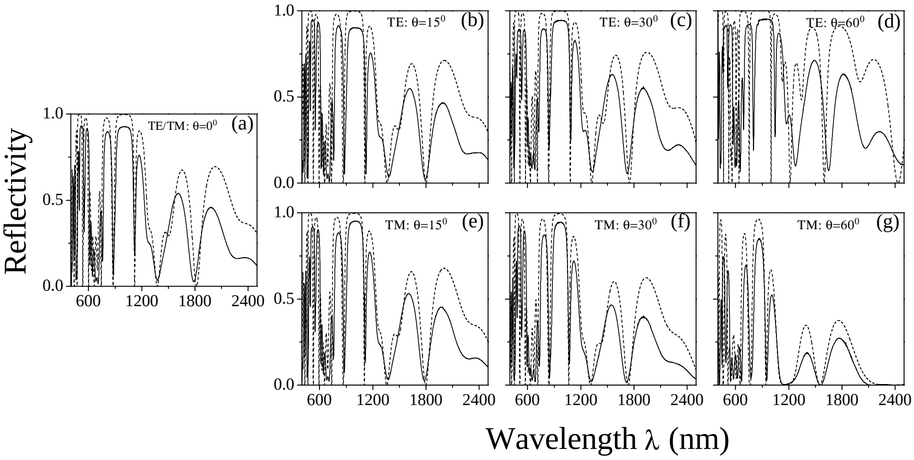

Fig. 4 – The measured and calculated reflectivity spectra of the Thue-Morse TiO_2/SiO_2 multilayer films S_5 for the transverse electric (TE) mode and the transverse magnetic (TM) mode at different incident angles θ. TE/TM mode: (a) $\theta = 0$; TE mode: (b) $\theta = 15°$, (c) $\theta = 30°$, and (d) $\theta = 60°$; TM mode: (e) $\theta = 15°$, (f) $\theta = 30°$, and (g) $\theta = 60°$. Note that, in each figure, the solid curve is for the measured spectrum, while the dashed curve for the calculated one.

and titanium dioxide are selected as dielectric materials A and B, respectively. Their refractive indexes are $n_A = 1.47$ (SiO_2) and $n_B = 2.30$ (TiO_2) for the wavelength of 700 nm. By electron-gun evaporation method, the Thue-Morse TiO_2/SiO_2 multilayer films were fabricated on the glass substrate. Before evaporation, vacuum of the chamber was better than 2×10^{-5} Torr. Thereafter, purified oxygen gas was introduced so the film was fabricated in an oxygen atmosphere. The pressure was 0.8×10^{-4} Torr for SiO_2 deposition and 2×10^{-4} Torr for TiO_2. The thickness of the film was controlled by quartz-crystal monitoring at a frequency of 5.0 MHz, and also the quarter-wave and half-wave optical thicknesses were optically monitored. The thicknesses of two materials in the film were chosen to satisfy $n_A d_A = n_B d_B$, which gives the same phase shift in the two materials, *i.e.*, $\delta_A = \delta_B = \delta$. The central wavelength was set to 700 nm which gives $d_A \approx (700\,\text{nm})/4n_A \approx 119.0\,\text{nm}$, and $d_B \approx (700\,\text{nm})/4n_B \approx 76.1\,\text{nm}$.

The optical reflectivity of the multilayer films has been measured by PerkinElmer Lambda 900 spectrophotometer in the range of wavelength from 400 nm to 2500 nm. The optical spectrum was calibrated relative to the reflectivity of silver. The measured and calculated reflectivity spectra of the Thue-Morse TiO_2/SiO_2 multilayer films S_5 are shown in fig. 4 at different incident angles, respectively. It can be seen that, with increasing the angle of incidence, both reflection bands of TE-polarized and TM-polarized light gradually shift to the shorter-wavelength region. In the case of TE wave, the bandwidth of reflection gradually increases if the incident angle increases (as shown in figs. 4(a)-(d)), while in the case of TM wave, the reflection bandwidth gradually decreases (as shown in figs. 4(a) and (e)-(g)). As a result, in the Thue-Morse TiO_2/SiO_2 multilayer, there is indeed the omnidirectional PBG for the TE mode, but no omnidirectional PBG for the TM mode. The reason is that the ratio of the refraction indexes of dielectric materials TiO_2 and SiO_2 is not large enough, which agrees with the calculated photonic band structure (shown in fig. 2(b)). The experimental spectra of reflectivity reasonably match the calculated ones (the deviation of the central wavelength is below 2%). Interestingly, such kind of Thue-Morse dielectric multilayers can be applied to separate the TE and TM modes in the optical device.

In summary, the omnidirectional photonic bandgaps have been investigated in the Thue-Morse dielectric multilayers. Due to the self-similarity of Thue-Morse structure, multiple PBGs exist both for the TE mode and for the TM mode. It is shown that the number and the width of omnidirectional PBG definitely depend on the ratio of the refraction indexes ($r = n_B/n_A$) and the thicknesses of the dielectric materials. If r is small, the omnidirectional PBG does not exist. With increasing r, the omnidirectional PBG appears for the TE mode, but not for the TM mode. If r is large enough, the omnidirectional PBG comes out for both TE and TM polarizations. Further by increasing r, multiple omnidirectional PBGs can be obtained. The theoretical result has been partly verified by the optical observation in Thue-Morse TiO_2/SiO_2 multilayers in the region of visible and near-infrared light. A similar feature can be expected in other aperiodic dielectric structures. The work provides a special mechanism for achieving the omnidirectional reflection in multiple frequency ranges, which may have potential applications of multichannel optical devices and all-dielectric waveguides. In addition, we expect that with this progress, the application of the dielectric reflection mirrors can be widened.

* * *

This work was supported by the grants from National Nature Science Foundation of China (10374042, 90201039, 10274029, 10021001), the State Key Program for Basic Research from the Ministry of Science and Technology of China, NSF of Jiangsu (BK2004209), and partially by the Ministry of Education of China and Fok Ying Tung Education Foundation.

REFERENCES

- [1] Yablonovitch E., *Phys. Rev. Lett.*, **58** (1987) 2059; John S., *Phys. Rev. Lett.*, **58** (1987) 2486.
- [2] Joannopoulos J. D., Meade R. D. and Winn J. N., *Photonic Crystals* (Princeton University Press, Princeton) 1995; Sokoulis C. M., *Photonic Band Gaps Localization* (Plenum, New York) 1993.
- [3] Joannopoulos J. D., Villeneuve P. and Fan S., *Nature (London)*, **386** (1997) 143; Scalora M., Dowling J. P., Bowden C. M. and Bloemer M. J., *Phys. Rev. Lett.*, **73** (1994) 1368; Temelkuran B. and Ozbay E., *Appl. Phys. Lett.*, **74** (1999) 486; Chen K. M., Sparks A. W., Luan H. C., Lim D. R., Wada K. and Kimerling L. C., *Appl. Phys. Lett.*, **75** (1999) 3805; Foulger S. H., Jiang P., Ying Y., Lattam A. C., Smith D. W. and Ballato J., *Adv. Mater.*, **13** (2001) 1898; Yang X. B., Liu Y. Y. and Fu X. J., *Phys. Rev. B*, **59** (1999) 4545.
- [4] Macia E., *Appl. Phys. Lett.*, **73** (1998) 3330; *Phys. Rev. B*, **63** (2001) 205421.
- [5] Peng R. W., Huang X. Q., Qiu F., Wang Mu, Hu A. and Jiang S. S., *Appl. Phys. Lett.*, **80** (2002) 6063; Peng R. W., Liu Y. M., Huang X. Q., Qiu F., Wang Mu, Hu A., Jiang S. S., Feng D., Ouyang L. Z. and Zou J., *Phys. Rev. B*, **69** (2004) 165109.
- [6] Fink Y., Winn J. N., Fan S., Chen C., Michel J., Joannopoulos J. D. and Thomas E. L., *Science*, **282** (1998) 1678.
- [7] Ye Y. H., Bader G. and Truong V., *Appl. Phys. Lett.*, **77** (2000) 235; Jiang Z. M., Shi B., Zhao D. T., Liu J. and Wang Xun, *Appl. Phys. Lett.*, **79** (2001) 3395; Wang X., Hu X., Li Y., Jia W., Xu C., Liu X. and Zi J., *Appl. Phys. Lett.*, **80** (2002) 4291; Bruyant A., Lerondel G., Reece P. J. and Gal M., *Appl. Phys. Lett.*, **82** (2003) 3227; Lee H. Y. and Yao T., *J. Appl. Phys.*, **93** (2003) 819.
- [8] Riklund R. and Severin M., *J. Phys. C*, **21** (1998) 3217; Dulea M., Severin M. and Riklund R., *Phys. Rev. B*, **42** (1990) 3680.
- [9] Godreche C. and Luck J. M., *J. Phys. A*, **23** (1990) 3769; Qin M. G., Ma H. R. and Tsai C. H., *J. Phys. Condens. Matter*, **2** (1990) 1059; Bovier A. and Ghez J., *J. Phys. A*, **28** (1995) 2313; Chattopadhyay S. and Chacrabarti A., *J. Phys. Condens. Matter*, **12** (2000) 5681.
- [10] Liu N. H., *Phys. Rev. B*, **55** (1997) 3543; Qiu F., Peng R. W., Huang X. Q., Liu Y. M., Wang Mu, Hu A. and Jiang S. S., *Europhys. Lett.*, **63** (2003) 853.
- [11] Pendry J. B. and Mackinnon A., *Phys. Rev. Lett.*, **69** (1992) 2772.