Resonant transmission of light waves in dielectric heterostructures

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We investigate the propagation of electromagnetic wave through dielectric heterostructures with transfer-matrix method. It is shown that if a dimerlike positional correlation (DPC) is introduced to the heterostructure, resonant transmission of light waves will definitely take place. The resonant transmissions are characterized by perfect transmission peaks in the photonic band gap, which is demonstrated by the electric-field distribution at corresponding resonant modes. The numerical calculations are in good agreement with the analytical predictions. Furthermore, by applying the SiO_2/Si heterostructure with DPC, resonant modes can appear within the photonic band gap at the telecommunication wavelengths of 1.55 and 1.3 μm. This finding is expected to have potential applications in wavelength division multiplexing system. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1929847]

I. INTRODUCTION

Motivated by the work of Yablonovitch[1] and John,[2] there has been increasing interest in the studies of dielectric materials with the photonic band gaps (PBGs), such as photonic crystals[3,4] and quasicrystals.[5] The propagation of photons with energies in PBGs is forbidden, which makes it possible to control photons in dielectric microstructures similar to manipulating electrons in solids. Up to now, great efforts have been devoted to the manipulation of PBGs in crystals in order to make photon a real alternative of electron as the information carrier.[6–8] Practically it is necessary to achieve a tunable PBG material to select photons of certain frequencies and to obtain high transmittivity at the desired frequencies.

In this paper, we report the optical transmission in a specially designed dielectric heterostructure. It is found that once the dimerlike positional correlation (DPC) exists in the structure, perfect transmission peaks appear in the PBG. The resonant modes can be expressed analytically. As an example, for the heterostructure of SiO_2/Si with DPC, resonant modes occur in the photonic band gap at the wavelengths of 1.55 and 1.3 μm, which are the wavelengths currently used for telecommunication. This feature may have potential application in the wavelength division multiplexing system.

II. THE THEORETICAL MODEL AND THE ANALYTICAL ANALYSIS

Consider the optical propagation through a dielectric multilayer $S_0=\{A_1A_2,\ldots,A_i,\ldots,A_{m-1}A_m\}$, where there are m dielectric layers $A_1,A_2,\ldots,A_i,\ldots,A_m$ with their refractive indices $\{n_i\}$ and thickness $\{d_i\}$, respectively. We use the transfer-matrix method, and follow the description of the electric field in the report of Kohmoto *et al.*[9] In the case of normal incidence and polarization parallel to multilayer surfaces, the transmission through the interface $A_j \leftarrow A_i$ is given by the transfer matrix

$$T_{i,j}=\begin{pmatrix}1 & 0\\ 0 & n_i/n_j\end{pmatrix}. \tag{1}$$

The light propagation within the layer A_i is described by matrix T_i,

$$T_i=\begin{pmatrix}\cos\delta_i & -\sin\delta_i\\ \sin\delta_i & \cos\delta_i\end{pmatrix}, \tag{2}$$

where $\delta_i=kn_id_i$ is the phase shift, $k=2\pi/\lambda$ is the vacuum wave vector, λ is the optical wavelength in the vacuum, and d_i is the thickness of the layer A_i. Therefore, the whole multilayer is represented by a product matrix M relating the incident and reflection waves to the transmission wave. The total transmission matrix M has the form

$$M=\begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}. \tag{3}$$

Using the unitary condition $\det|M|=1$, the transmission coefficient of the multilayer can be written as

$$\breve{T}=\frac{4}{\sum_{i,j=1}^{2} m_{ij}^2+2}. \tag{4}$$

Now we consider a dielectric heterostructure with two substructures. By defining the right substructure as $S_R=S_0$ (S_0 is described above) and the left substructure as $S_L=\{A'_mA'_{m-1},\ldots,A'_i,\ldots,A'_2A'_1\}$, we construct the dielectric

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heterostructure S:

$$S = S_L \cup S_R = \{\underbrace{A'_m A'_{m-1}, \ldots, \underbrace{A'_i, \ldots, \underbrace{A'_2 \underbrace{A'_1 | A_1} A_2}, \ldots, A_i}, \ldots, A_{m-1} A_m}\}. \tag{5}$$

The total transmission matrix through the heterostructure S can be represented by

$$M = \{\underbrace{T_{m'} T_{m'-1,m'}, \ldots, \underbrace{T_{i',i'+1} T_{i'} T_{i'-1,i'}, \ldots, \underbrace{T_2, T_{1',2'}, \underbrace{T_{1'}\} \cdot \{T_1} T_{2,1}, T_2}, \ldots, T_{i,i-1} T_i T_{i+1,i}}, \ldots, T_{m,m-1} T_m}\}. \tag{6}$$

In order to show that DPC may induce the resonant transmission, we consider the simple setting: (i) The dielectric layers A_i and A'_i are the same dielectric material, that is to say, they have the same refractive index n_i, but their layer thicknesses are d_i and d'_i, respectively. (ii) In the right substructure S_R, the phase shift is identical, i.e., $\delta_{Ai} = (2\pi/\lambda_R) n_i d_i \equiv \delta_R$ $(i=1,2,\ldots,m)$; While in the left substructure S_L, the phase shift is identical too, i.e., $\delta_{Ai'} = (2\pi/\lambda_L) n_i d'_i \equiv \delta_L$ $(i=1,2,\ldots,m)$. This condition can be experimentally satisfied by tuning the thickness of each dielectric layer. Now looking at the center of M [shown in Eq. (6)], the pair of matrices related to the dimer $A'_1 A_1$ is

$$M_1 = T_{1'} T_1 = \begin{pmatrix} \cos \delta_c & -\sin \delta_c \\ \sin \delta_c & \cos \delta_c \end{pmatrix}, \tag{7}$$

where $\delta_c = \delta_L + \delta_R$. If we define the central wavelengths of substructures as $\lambda_R = 4 n_i d_i$ in S_R and $\lambda_L = 4 n_j d'_j$ in $S_L (i,j = 1,2,\ldots,m)$, we have $\delta_c = [\pi(\lambda_L + \lambda_R)]/2\lambda$, where λ is the wavelength of incident light in the vacuum. Obviously in the case of

$$\lambda = (\lambda_L + \lambda_R)/2p, \tag{8}$$

we have

$$M_1 = (-1)^p I,$$

where I is a unit matrix and p is an integer. Meanwhile, the most central part of M is simplified as the product of matrices $T_{1',2'} \cdot T_{2,1}$, which is again a unit matrix, i.e., $T_{1',2'} \cdot T_{2,1} = I$. On this basis, the second pair of matrix $T_{2'} T_2$ [which corresponds to the dimer $A'_2 A_2$ in Eq. (5)] comes to the center of M, as shown in Eq. (6). Again, if $\lambda = (\lambda_L + \lambda_R)/2p$ is satisfied, $M = T_{2'} T_2 = (-1)^p I$ holds. Repeating the same pairing procedure, and following the rules of $T_{i',j'} \cdot T_{j,i} = I$ and $M_j = T_{j'} T_j = (-1)^p I$ (which corresponds to the jth pair of dimer $A'_j A_j$), the total transfer matrix through the heterostructure S is

$$M = (-1)^{m \cdot p} I. \tag{9}$$

According to Eq. (4), the transmission coefficient is

$$\breve{T}(S) = 1. \tag{10}$$

It can be clearly seen that the dimers $A_{j'} A_j$ in the dielectric heterostructure can eventually lead to perfect resonant transmission.

Now we focus on the resonant modes in the dielectric heterostructures with DPC. First, according to the above analysis, the resonant transmission will happen at the wavelengths satisfying $\lambda = (\lambda_L + \lambda_R)/2p$ (p is an integer). It is easy to prove the following special cases. (i) The resonant transmission will happen at the wavelength of λ_R if $\eta = 1/(2q - 1)$, where η is the ratio of the central wavelength in two substructures, i.e., $\eta = \lambda_R/\lambda_L$, and q is an integer. (ii) The resonant transmission will happen at the wavelength of λ_L if $\eta = 2q - 1$. Second, we consider the case that the substructure in the dielectric heterostructure also has a DPC, for example, the two substructures possess mirror symmetry. Carrying out the similar analysis in the substructure as shown in Eqs. (5)–(10), we find that the resonant modes appear at the wavelengths satisfying $\lambda = (\lambda_R + \lambda_L)/2p$ or $\lambda = \min\{\lambda_R, \lambda_L\}$, in the case that the ratio of the central wavelengths in two substructures is an integer, i.e., $\eta = q$. Here $\min\{\lambda_R, \lambda_L\}$ represents the minimum of λ_R and λ_L. Repeating the same operation, we can make the subsubstructure a DPC, and so on. Finally, it can be expected that more and more resonant modes generate in the dielectric heterostructures. In this way, perfect transmissions can be designated at specific wavelengths.

III. THE NUMERICAL CALCULATIONS

Based on Eqs. (1)–(4), the transmission of light waves in the dielectric heterostructure can be numerically calculated. In order to exhibit the geometrical effect in the dielectric heterostructure with DPC, we give a simple setting in the following calculation. We select two dielectric materials, silicon dioxide (SiO_2) and silicon (Si). Silicon dioxide is used as dielectric material A (or A') and silicon as dielectric material B (or B'). Their refractive indices are $n_{A(A')} = n_{SiO_2} = 1.46$ and $n_{B(B')} = n_{Si} = 3.5$, respectively.

The dielectric heterostructure with two substructures is constructed as follows. Firstly, we define the left substructure as $G_n = (AB)^n$, where n is the repeated number of AB, and the right substructure as $H_n = (B'A')^n$. Then, the dielectric heterostructure can be expressed as $U_n = G_n \cup H_n$. Here both A

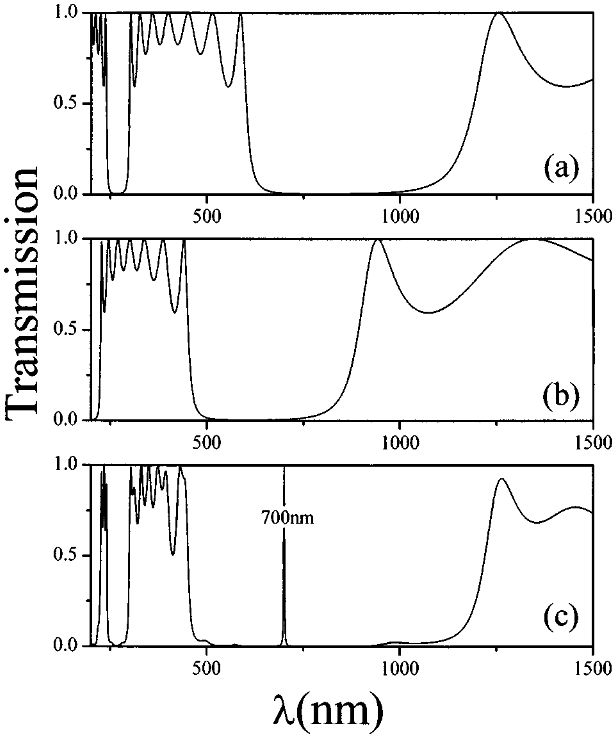

FIG. 1. The transmission coefficient T as a function of the optical wavelength λ for the SiO_2/Si multilayers with the structure as (a) the periodic structure H_4 (8 layers); (b) the periodic structure G_4 (8 layers); and (c) the heterostructure $U_4=G_4\cup H_4$ (16 layers). Here the central wavelength of the substructure is $\lambda_R=800$ nm in H_4 and $\lambda_L=600$ nm in G_4.

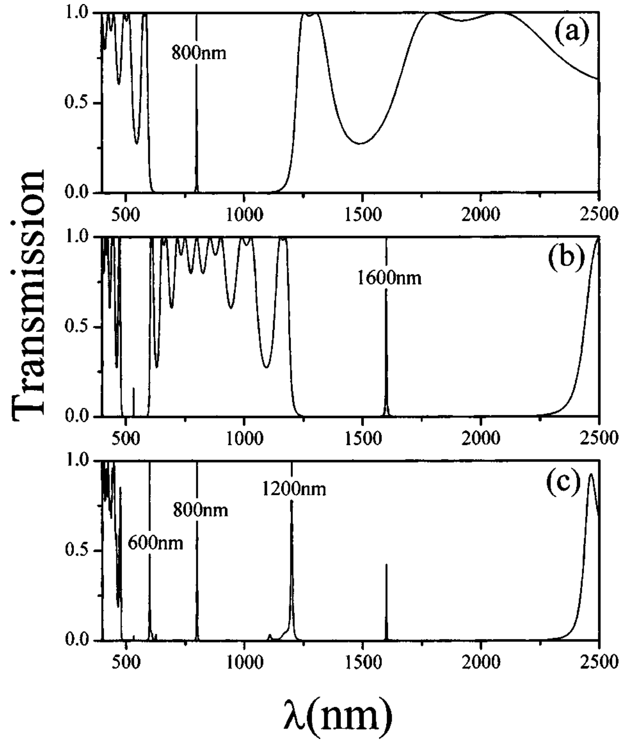

FIG. 2. The transmission coefficient T as a function of the optical wavelength λ for the SiO_2/Si multilayers with the heterostructure as (a) SH_4 (16 layers); (b) SG_4 (16 layers); and (c) $SU_4=SG_4\cup SH_4$ (32 layers). Here the central wavelength of the substructure is $\lambda_R=800$ nm in SH_4 and $\lambda_L=1600$ nm in SG_4.

and A' are silicon dioxide (SiO_2), and both B and B' are silicon (Si). The thicknesses of these dielectric materials are $d_{A(B)}=\lambda_L/4n_{A(B)}$ in material A (or B) and $d_{A'(B')}=\lambda_R/4n_{A'(B')}$ in material A' (or B'), respectively. λ_L and λ_R are the central wavelengths in the substructures. According to Eq. (5), DPC can be identified in U_n. For example, in the case of $n=4$, we have

$$G_4=\{ABABABAB\},$$

$$H_4=\{B'A'B'A'B'A'B'A'\}, \tag{11}$$

$$U_4=G_4\cup H_4=\{\mathrm{ABABABAB}\|\mathrm{B'A'B'A'B'A'B'A'}\},$$

Figures 1(a)–1(c) show the transmission coefficients as a function of optical wavelength in the SiO_2/Si films with the substructures H_4 and G_4, and the heterostructure U_4, respectively. The central wavelength of the substructure is $\lambda_L=600$ nm for G_4 and $\lambda_R=800$ nm for H_4. It is obvious that the dielectric heterostructure can enlarge the photonic band gap (PBG) (as shown in Fig. 1). The PBG of the heterostructure U_4 covers the wavelength from 500 to 1050 nm [shown in Fig. 1(c)], which is wider than the PBG in G_4 (500–760 nm) and that in H_4 (650–1020 nm). The perfect transmission peak appears exactly at $\lambda=(\lambda_L+\lambda_R)/2=700$ nm in the PBG of the dielectric heterostructure U_4. This resonant mode originates from the dimers AA' and BB' in U_4, as discussed in Sec. II.

More resonant transmissions will occur in the dielectric heterostructure if the substructure also possesses a DPC. We define the left substructure as $SG_n=G_n\cup G_n^{-1}=(AB)^n(BA)^n$ and the right substructure as $SH_n=H_n\cup H_n^{-1}=(A'B')^n(B'A')^n$. Then we obtain the following heterostructure $SU_n=SG_n\cup SH_n$, in which two substructures have a DPC, i.e., a mirror symmetry. As an example, we have

$$\mathrm{SG}_4=\{\mathrm{ABABABAB}\|\mathrm{BABABABA}\},$$

$$\mathrm{SH}_4=\{\mathrm{A'B'A'B'A'B'A'B'}\|\mathrm{B'A'B'A'B'A'B'A'}\},$$

$$\begin{aligned} SU_4 &= SG_4\cup SH_4 \\ &=\{ABABABAB|BABABABA\| \\ &\quad A'B'A'B'A'B'A'B'|B'A'B'A'B'A'B'A'\} \end{aligned} \tag{12}$$

for the case of $n=4$. Figures 2(a)–2(c) show the transmission coefficients as a function of the optical wavelength in the SiO_2/Si films with the substructures SH_4 and SG_4 and the heterostructure SU_4, respectively. The central wavelength of substructure is $\lambda_L=1600$ nm in SG_4 and $\lambda_R=800$ nm in SH_4. As shown in Fig. 2(c), perfect transmission peaks in the PBG of the heterostructure $SU_4=SG_4\cup SH_4$ exist at the following wavelengths: (i) $\lambda_1=(\lambda_L+\lambda_R)/(2p)=(800+1600\text{ nm})/2=1200$ nm, where $p=1$. (ii) $\lambda_2=(\lambda_L+\lambda_R)/(2p)=(800+1600\text{ nm})/4=600$ nm, where $p=2$. As discussed in Sec. II, these two resonant modes come from DPC between the left substructure SG_4 and the right one SH_4 (iii) $\lambda_3=\min(\lambda_L,\lambda_R)=\min(1600,800\text{ nm})=800$ nm. This mode comes from DPC in the substructure SH_4. The numerical calculation is indeed in good agreement with the analytical analysis in Sec. II.

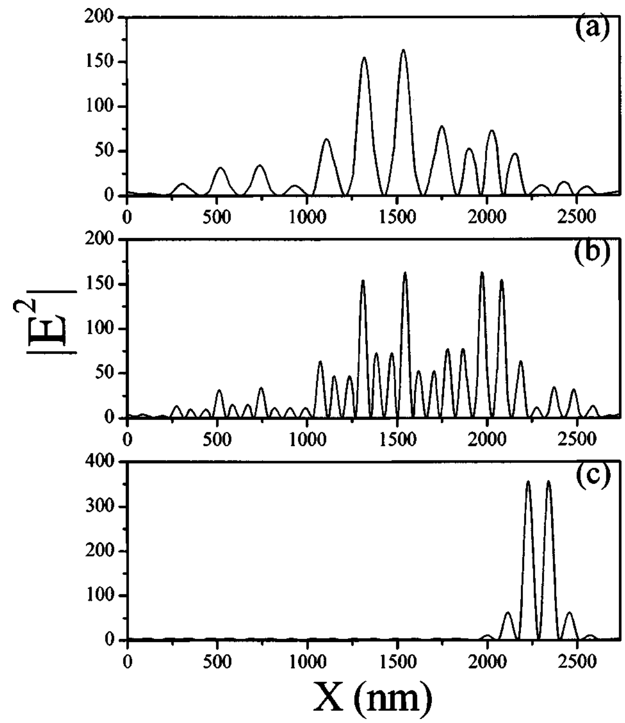

FIG. 3. The electric-field distributions in the SiO_2/Si heterostructure $SU_4 = SG_4 \cup SH_4$ at the resonant mode: (a) $\lambda_1 = 1200$ nm, (b) $\lambda_2 = 600$ nm, and (c) $\lambda_3 = 800$ nm, respectively.

In order to demonstrate the resonant transmissions in the heterostructure with DPC, we have calculated the electrical field distribution in the structure. Figure 3 shows the electric-field distribution in the SiO_2/Si heterostructure $SU_4 = SG_4 \cup SH_4$ at three resonant modes $\lambda_1 = 1200$ nm, $\lambda_2 = 600$ nm, and $\lambda_3 = 800$ nm, which have been described above. It is found that the electric fields of the modes $\lambda_1 = 1200$ nm and $\lambda_2 = 600$ nm are almost extended through the whole heterostructure SU_4 [as shown in Figs. 3(a) and 3(b)]. The electric field of the mode $\lambda_3 = 800$ nm locates at the right substructure SH_4 [as shown in Fig. 3(c)]. Therefore, the resonant modes λ_1 and λ_2 indeed originate from the positional correlation in the whole structure SU_4, and the resonant mode λ_3 from the positional correlation in the right substructure SH_4.

IV. RESONANT MODES AT THE WAVELENGTHS FOR TELECOMMUNICATION

According to the above analysis, the resonant transmission takes place when DPC exists in the dielectric heterostructure. The resonant transmission can be tuned to the specific wavelength by introducing special "dimers" in the structure. To obtain the high-quality perfect transmissions at the wavelengths for telecommunication, we construct the following heterostructures:

$$SV_1 = (AB)^2AA(BA)^2 \| (A'B')^2A'A'(B'A')^2, \tag{13}$$

$$SV_2 = (AB)^3(BA)^3 \| (A'B')^3(B'A')^3,$$

$$SV_3 = (AB)^3AA(BA)^3 \| (A'B')^3A'A'(B'A')^3,$$

$$SV_4 = (AB)^4(BA)^4 \| (A'B')^4(B'A')^4.$$

Figure 4 shows the transmission spectra of SiO_2/Si films with the structures as SV_1, SV_2, SV_3, and SV_4, respectively. The central wavelength is $\lambda_L = 1550\ \text{nm} \times 2 - 1300\ \text{nm} = 1800$ nm in the left substructure and $\lambda_R = 1300$ nm in the right substructure. As shown in Figs. 4(a)–4(d), within the same PBG, resonant transmissions take place at the wavelengths of 1.55 and 1.30 μm, respectively. The transmittivity and the quality factor of these two transmission peaks increase gradually as increasing the layer number in the heterostructure. The quality factors Q are as high as 680 at the mode of $\lambda = 1.30$ μm and 2512 at the mode of $\lambda = 1.55$ μm in the heterostructure SV_4 [as shown in Fig. 4(d)]. These two wavelengths are exactly the ones currently used for telecommunication.

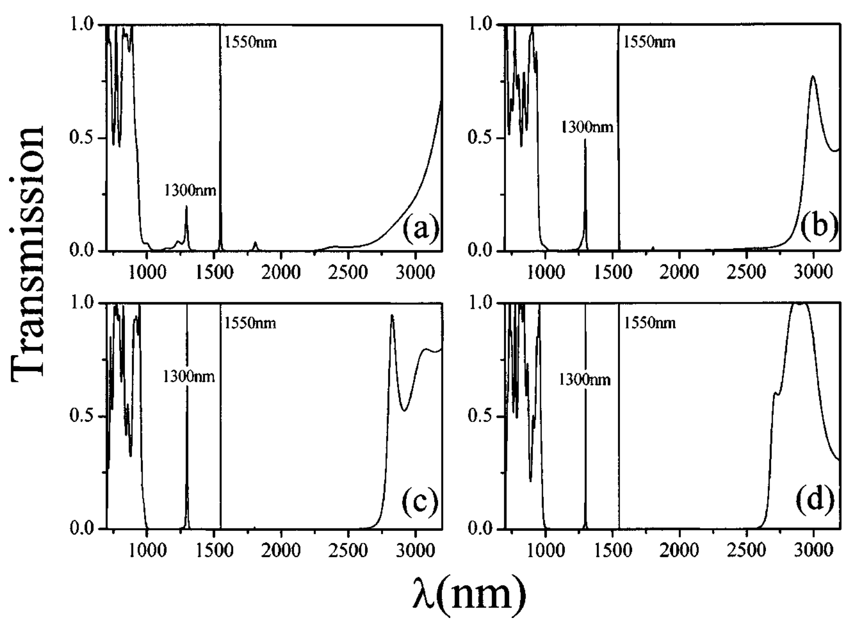

FIG. 4. The transmission coefficient T as a function of the optical wavelength λ for the SiO_2/Si multilayers with the heterostructure as: (a) SV_1, (b) SV_2, (c) SV_3, and (d) SV_4, respectively. The central wavelength is $\lambda_L = 1800$ nm in the left substructure and $\lambda_R = 1300$ nm in the right substructure. The resonant transmissions are found at the telecom wavelengths of 1.55 and 1.30 μm within the same photonic band gap.

V. SUMMARY

To summarize, we have investigated the optical propagation in the dielectric heterostructure with transfer-matrix method. It is shown that DPC can indeed induce resonant transmission of light waves in the heterostructure. The resonant transmissions are characterized by the perfect transmission peaks in the photonic band gap, and demonstrated by the electric-field distribution at corresponding resonant modes. Furthermore, we show that the resonant modes can be obtained at the telecom wavelengths of 1.55 and 1.3 μm within the same photonic band gap of SiO_2/Si heterostructure. This work demonstrates a way to tune the photonic band gap and the resonant modes therein, and may have potential applications in the optoelectronic devices such as the wavelength division multiplexing system.

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